Pricing and Distributed Power Control in Wireless Relay Networks

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***Abstract*—In this paper, we consider a wireless amplify-and-forward relay network with one relay node and multiple source-destination pairs/users and propose a pricing framework that enables the relay to set prices to maximize either its revenue or any desirable system utility. Specifically, depending on the quality of the received signals, the relay sets prices and correspondingly charges the users utilizing its resources for their transmissions. The price is determined in such a way that the relay's revenue or system utility is maximized. Given the specified price, the users competitively employ the relay node to forward their signals. We model each user as a rational player, which aims at maximizing its own net utility through power allocation, and analyze the competition among the users within the framework of noncooperative game theory. It is shown that, in the game played by the users, there always exists a unique pure Nash equilibrium point that can be achieved through distributed iterations. Next, subject to the availability of complete information about the users at the relay, we propose a low-complexity uniform pricing algorithm and an optimal differentiated pricing algorithm, in which the relay either charges the users at a suboptimal uniform price or charges different users at different prices. We also show that, by applying the differentiated pricing algorithm that enforces the users to transmit at certain power levels, any system utility can be maximized. Extensive simulations are conducted to quantify the performance of the proposed methods.**

***Index Terms*—Distributed power allocation, game theory, interference channel, relay networks.**

I. Introduction

FOR many wireless networks, the transmission between two distant users may have to be accomplished with the help of an intermediate node, i.e., relay, due to transmit power or other constraints [1]. In the presence of a relay node, distributed spatial diversity, or cooperative diversity, can be created without physically packing multiple antennas into small-size nodes as long as certain signal combining techniques are applied at the destination [2], [3].

The traditional network resource allocation largely relies on system-wide centralized management, which requires all the users to cooperatively follow the resource sharing mechanism and incurs a heavy spectral loss due to the signalling overhead associated with the information exchange and coordination. Nevertheless, in the absence of a central controller, rational or selfish users have incentives to optimize their own performances independently, without considering the social welfare and thus, the existing centralized mechanisms are no longer applicable in such settings. An alternative solution is to model a network of selfish users using noncooperative game theory [35]. Furthermore, it has been demonstrated in the literature that appropriate pricing techniques can be deployed among multiple selfish users to implement various resource allocation policies, including, but not limited to, revenue maximization [9], social-welfare improvement [18], user fairness guarantee [20] and system-wide optimization [19]. Interested readers are referred to [8] for a survey on game-theoretic resource allocation and pricing mechanisms. In wireless relay networks, without a proper compensation framework, relays have no incentives to forward the signals of various users to the corresponding destinations, since this is done at the expense of their own energy consumption. Hence, pricing becomes a useful and efficient mechanism that reimburses relays for using their resources by making payments,[1] thereby providing the relays with incentives to forward the other users' signals [11]–[13].

In this paper, we focus on a wireless relay network, in which there exists one relay node and multiple source-destination pairs/users.[2] We propose a pricing mechanism that gives the relay incentives to forward the users' signals to the destinations. In particular, the price is determined by the relay such that its revenue[3] or system utility is maximized. Given the specified price, the users competitively utilize the relay node to forward their signals and make appropriate payments to the relay based on the receive signal-to-interference-plus-noise ratio (SINR). We model each user as a selfish player, which aims at maximizing its own net utility by adjusting its transmit power, and analyze the emerging competition among the users using noncooperative game theory. Specifically, given the knowledge of its local channel state information (CSI), each user maximizes its utility by optimally choosing its power level in response to the power allocation strategies of the other users. This process iterates until convergence. We show that, in the

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[1]The payments can be tokens, virtual money, etc., which can be used in the future by the relay to purchase resources from the other nodes in the network.

[2]Throughout this paper, we interchangeably use the term "user" to represent the source-destination pair.

[3]The dedicated relay incurs a fixed cost, e.g., power consumption, associated with forwarding the users' signals, and moreover, the relay's resource in the current time slot cannot be reserved for further use [17]. For instance, the cost of deploying the relay station and power expenditure is paid in advance by the infrastructure manager. Therefore, as described in the transmission protocol, the relay will forward the users' signals and revenue maximization is virtually equivalent to profit maximization [25], [27], [34].

noncooperative game played by the users, there always exists a unique pure Nash equilibrium point (NEP) that can be achieved through the distributed iterative power allocation process. Next, we assume that the relay has only incomplete information about the users (i.e., the number of users and the sum SINR when all the users transmit with their maximum powers) and propose a low-complexity uniform pricing algorithm based on which the relay charges the users at a suboptimal uniform price. Subsequently, we extend the uniform pricing algorithm to differentiated pricing by assuming that the relay has complete information about the users (i.e., channel coefficients, power constraints, etc.). Furthermore, we show that, by utilizing the differentiated pricing algorithm, any system utility can be maximized even though the users behave selfishly. Finally, extensive simulations are conducted to verify the performance of the proposed methods.

The main contributions of this paper are threefold: (i) we focus on a relay network with multiple users that are modeled as selfish players competing against each other for the scarce network resource, i.e., the relay, and study the NEP of the noncooperative game; (ii) depending on how much information the relay has about the users, we propose two pricing algorithms, i.e., uniform pricing with incomplete information and differentiated pricing with complete information; (iii) the proposed differentiated pricing algorithm enforces the users to transmit at desired power levels at the NEP and hence, can be applied to optimize any system utility, which includes the relay's revenue as a special case.

The rest of this paper is organized as follows. Section II provides the literature review and Section III describes the system model and problem formulation. In Section IV, a distributed power allocation algorithm along with two pricing algorithms are developed for the considered relay network. Simulation results are shown in Section V. Finally, concluding remarks are offered in Section VI.

II. Related Works

Power allocation, both with and without pricing, has been extensively studied in wireless networks. Next, we present a brief overview of the related works and describe the relationship to our proposed mechanism.

It is worth noting that pricing mechanisms, which originate from the competitive market theory [34], have been widely applied in the context of cognitive systems [9], [10] and relay networks [11]–[13]. For instance, given a wide-band uplink cognitive system, [9] proposes a differentiated pricing algorithm that charges different secondary users at different prices to maximize the revenue of the service provider. To utilize the benefits of distributed spatial diversity and guarantee incentive compatibility in wireless cooperative networks, [10] adopts the hierarchical Stackelberg game-theoretic framework where the primary user, as the leader, selects some secondary users as the cooperative relay nodes and, in return, grants the spectrum usage to the participating secondary users for their own data transmissions. As followers, the secondary users decide the payment made to the primary user to gain the channel access time and maximize their own utilities. Considering a cooperative network with multiple self-interested relays, the authors in [11] cast the problem of distributed power control and relay selection into the Stackelberg formulation. In particular, the relays are regarded as leaders that selfishly set the prices such that they can maximize the revenue. The payment made by the user serves as a reimbursement that gives the relays an incentive to participate in the cooperation. Similar compensation frameworks enabling the relay to forward the users' signals are proposed in the literature, e.g., [12] and [13].

Following a joint user-centric and network-centric optimization approach, the authors in [14] propose a distributed power control and revenue optimization framework in conventional cellular networks. Specifically, the network controller, e.g., base station, charges each user in accordance with its throughput while the users transmit over an interference channel and maximize the energy efficiency. The same approach is later applied in the multicell scenario [15]. In [16], an auction-based spectrum sharing protocol is proposed such that each user submits an optimal bid to the network manager to maximize the utility minus the payment. Two payment rules, i.e., SINR and power, are considered and it is shown that, with logarithmic utilities, the power auction outperforms the SINR auction in terms of the revenue from the network perspective. The auction framework is also extended in [17] to a cooperative network setting wherein the relay and the users are modeled as the auctioneer and bidders, respectively. Focusing on the classic Gaussian interference channel, [24] introduces the notion of "taxation" which captures the effect of one user's power allocation on the others', and presents a modified iterative water-filling algorithm that maximizes the sum utility. For a cellular network, the authors in [26] proposed a differentiated pricing mechanism such that any near-optimal system utility can be achieved.

In contrast with the existing literature, we shift our attention to a relay network with multiple selfish users and propose a pricing mechanism that can maximize the system utility and provide the relay with incentives to forward the signals of the users. New challenges emerging in such relay networks include: (i) how to design a proper pricing mechanism and how to set the price; (ii) given the price, how to model and analyze the competition among the selfish users; (iii) in view of the users' selfishness, how to maximize the system utility. In this paper, the users competitively adjust their transmit powers and utilize the relay node to accomplish their own transmissions and, as the service provider, the relay charges all the users according to either uniform or differentiated pricing algorithm to maximize the revenue. Furthermore, the differentiated pricing can be applied to optimize any system utility.

III. System Model

Consider a wireless relay network consisting of one relay node and multiple source-destination pairs.[4] The sources and destinations are indexed by $\mathcal{S}_i$ and $\mathcal{D}_i$, respectively, for $i = 1, 2 \cdots Q$, and the relay node is represented by $\mathcal{R}$. Similar system models with one relay node and multiple users have

[4]As in [4], [5], the analysis throughout this paper can be applied to a network with more than one relays, provided that the network can be classified into multiple clusters, each of which consists of one relay and multiple users, and different clusters are transmitting over different channels.

been considered in the literature for different purposes as well [4]–[7], wherein [4]–[6] focus on orthogonal transmissions without interferences and [7] studies interference management in a two-way relay channel [1].

A. Network Model

We assume that the channels are flat (or frequency nonselective) fading. When the channels are frequency-selective fading and divided into multiple subchannels (e.g., OFDM), the proposed algorithm in this paper can still be applied on a per-subchannel basis if each user has an individual maximum power constraint for each subchannel. Nevertheless, if each user has a total power constraint across all the subchannels, it is intrinsically difficult to generalize the proposed algorithm. In order to keep the analysis tractable, we note that it is a common practice to focus on a flat fading (or frequency nonselective fading) channel model when studying pricing-related algorithms ([11], [14], [17], [26]). The channel coefficients for the $\mathcal{S}_i - \mathcal{R}$ and the $\mathcal{R} - \mathcal{D}_i$ channels are denoted by g_i and h_i, respectively, for $i = 1, 2 \cdots Q$. The transmit powers of $\mathcal{S}_i$ and $\mathcal{R}$ are p_i and $p_{\mathcal{R}}$, respectively. The local CSI, i.e., g_i and h_i, is only obtained by user i, and neither g_j nor h_j is known to user i, if $j \neq i$, due to the distributed nature of the considered communication problem. Furthermore, we assume that the zero-mean complex additive white Gaussian noise (AWGN) at each node to has a variance[5] of N_0. Due to the half-duplex constraint, we consider orthogonal relaying transmissions, e.g., the source nodes and the relay node transmit in two nonoverlapping time slots. The direct link between $\mathcal{S}_i$ and $\mathcal{D}_i$ is neglected due to, for instance, the shadowing effects [1]. To forward the data from the source to the destination, we adopt the classical amplify-and-forward strategy [3] as the relaying operation, which has been shown to be an appealing technique due to its low cost and easy implementation as compared to the decode-and-forward protocol [30]. Hence, the signals received at $\mathcal{R}$ and $\mathcal{D}_i$ can be written, respectively, as

$$y_{\mathcal{R}} = \sum_{j=1}^{Q} g_j \sqrt{p_j} x_j + n_{\mathcal{R}} \text{ and } y_i = \alpha h_i y_{\mathcal{R}} + n_i, \qquad (1)$$

where x_i is the unit-variance transmit signal from $\mathcal{S}_i$ to $\mathcal{D}_i$, α is the amplification factor of $\mathcal{R}$, $n_{\mathcal{R}}$ and n_i are the statistically-independent AWGN terms at $\mathcal{R}$ and $\mathcal{D}_i$, respectively. The amplification factor

$$\alpha = \sqrt{\frac{p_{\mathcal{R}}}{\sum_{j=1}^{Q} |g_j|^2 p_j + N_0}}$$

which is public information available to all the users, is chosen to satisfy the fixed power constraint at the relay. Assuming that $\mathcal{D}_i$ is only interested in the signal x_i and treats the multiuser interference as noise [23], [29], we can then express the receive SINR at $\mathcal{D}_i$ as

$$\gamma_i = \frac{|g_i|^2 |h_i|^2 p_{\mathcal{R}} p_i}{|g_i|^2 N_0 p_i + (|h_i|^2 p_{\mathcal{R}} + N_0) \cdot \Delta_i} \qquad (2)$$

where $\Delta_i = \sum_{j=1, j\neq i}^{Q} |g_j|^2 p_j + N_0$. Recall that in an amplify-and-forward relay network with only one source node, only AWGN noise is amplified and forwarded by the relay to the destination node in addition to the desired signal component, and thus, the received signal-to-noise ratio (SNR) is expressed as

$$\frac{|g_i|^2 |h_i|^2 p_{\mathcal{R}} p_i}{|g_i|^2 N_0 p_i + (|h_i|^2 p_{\mathcal{R}} + N_0) N_0}.$$

When there are multiple source nodes transmitting simultaneously to the relay, both AWGN noise and multiuser interference (i.e., $\sum_{j=1, j\neq i}^{Q} |g_j|^2 p_j$) are amplified and forwarded to the destination nodes and hence, the resulting SINR expression becomes (2). The achievable rate of user i is therefore given by

$$R_i(p_i; p_{-i}) = \frac{1}{2} \log(1 + \gamma_i), \qquad (3)$$

where the scaling factor 1/2 is due to the fact that $\mathcal{S}_i$ transmits x_i only for half of the frame, γ_i is given in (2), and $p_{-i} = (p_1 \cdots p_{i-1}, p_{i+1} \cdots p_Q)$ is the vector of power allocation strategies of all the users except for user i.

Before proceeding to the problem formulation, we briefly discuss how transmissions using the relay node considered in this paper significantly differ from conventional single-hop transmissions [23], [24], despite the absence of direct channels. First, the signals are transmitted through a cascaded channel, i.e., multiaccess channel followed by broadcast fading channel. Second, the signal forwarded by the relay node is not "clean," whereas the source transmits noiseless signals to the destination in single-hop Gaussian interference channels, i.e., the relay amplifies the Gaussian noise, in addition to the desired signal, which can be seen from the signal model in (1). Hence, the analysis in this paper can be regarded as a generalization of the existing results on one-hop interference channels. As a special case, if the relay-destination channel is sufficiently good (i.e., $|h_i|^2 \to \infty$), the dual-hop relay channel reduces to the conventional multiaccess interference channel and the receive SINR of user i becomes

$$\gamma_i = \frac{|g_i|^2 p_i}{\sum_{j=1, j\neq i}^{Q} |g_j|^2 p_j + N_0}$$

which can also be obtained by taking the limit of (2) with respect to $|h_i|^2 \to \infty$.

B. Problem Formulation

There are various payment rules in communications networks. For instance, each individual user may be charged in proportion to the relay's transmit power [11], its throughput [14], receive SINR [16], [17], allocated rate [25], and its own

[5]This assumption is imposed only for the convenience of notation, as in [24], and can be relaxed without affecting the analysis in this paper.

transmit power [26]. In the problem considered in this paper, it is clear from (2) that the receive SINR is partially determined by the relay's power. Furthermore, it is the SINR that measures the quality of the received signal and thus influences the utility of each user. Hence, it is reasonable to assume that the payment made to the relay is a function of the receive SINR. In particular, we assume in this paper that the payment that user i needs to make to the relay is defined as $\pi_i\gamma_i$, where π_i is the price for user i set by the relay. As will be shown in this paper, this payment rule allows the relay to set optimal differentiated prices such that any system utility function can be maximized. Moreover, the considered payment rule charges each user in proportion to its receive SINR, and has been applied in [16] and [17] for different purposes (e.g., to achieve different tradeoffs between fairness and efficiency in a multiuser relay network [17]). Other similar payment rules can be found in [9], [14]. In general, the utility function is increasing and concave in the receive SINR [16], [17]. As a particular example and to gain more insights on the pricing algorithms, we adopt in the sequel the achievable rate[6] $R_i(p_i;p_{-i})$ as the utility function of user i. Given the utility function and payment rule, the payoff, or net utility function, of user i can therefore be expressed as the following surplus [9], [11], [17]

$$u_i(p_i;p_{-i}) = \frac{1}{2}\log(1+\gamma_i) - \pi_i\gamma_i, \tag{4}$$

where the first term $\frac{1}{2}\log(1+\gamma_i)$ is the achievable rate of user i, and $\pi_i\gamma_i$ is the payment made to the relay. From the perspective of the relay, in order to maximize the revenue collected from the users, the relay needs to set an optimal price vector $\mathbf{\Pi}^* = \{\pi_1^*, \pi_2^* \cdots \pi_Q^*\}$ such that

$$\mathbf{\Pi}^* = \arg\max_{\mathbf{\Pi}\succeq\mathbf{0}} \left(\sum_{i=1}^{Q} \pi_i\gamma_i(p_i;p_{-i})\right). \tag{5}$$

Note that the relay's price and the users' power allocation are coupled in a complex way, and we shall address the coupling in Section IV. In particular, relay's price influences the users' power allocation which, in turn, affects the relay's revenue. While we first use the revenue as a particular utility for the relay, we note that the proposed pricing mechanism can also be applied to maximize any system utility, making the proposed pricing framework a suitable option for managing wireless relay networks with selfish users.

IV. User-Centric Optimization and Pricing

In this section, we cast the user-level problem of distributed power allocation into the framework of noncooperative game theory. Adopting revenue as the relay's utility, we propose two pricing algorithms, i.e., uniform pricing and differentiated pricing. Then, we show that the differentiated pricing algorithm can maximize any system utility by enforcing the users to transmit at desired power levels.

A. Distributed Power Allocation

Noncooperative game theory is an effective tool to capture the selfish behaviors of self-interested players [35]. Given the price set by the relay, we can mathematically characterize the competition among the self-interested users as a noncooperative game

$$\mathcal{G}_{\text{user}} = \{\Omega, \{\mathcal{P}_i\}_{i\in\Omega}, \{u_i(p_i;p_{-i})\}_{i\in\Omega}\} \tag{6}$$

where $\Omega \triangleq \{1, 2\cdots Q\}$ is the set of active users (i.e., $\mathcal{S}_i - \mathcal{D}_i$ pair), $\mathcal{P}_i$ is the set of admissible power allocation strategies of user i defined as $\{p_i : 0 \le p_i \le p_i^{\max}\}$ and $u_i(p_i;p_{-i})$ is the payoff of user i given in (4). The optimal power of user i in response to the power levels of all the other users is referred to as the *best response* function denoted by $\mathcal{B}_i(p_{-i})$. In the noncooperative game played by the users, the NEP is achieved when user i, given p_{-i}, cannot increase its net utility $u_i(p_i;p_{-i})$ by unilaterally changing its own power p_i, for all $i \in \Omega$. Mathematically, the NEP, denoted by $\mathbf{p}^* = (p_1^*, p_2^* \cdots p_Q^*)$, of the user-level game $\mathcal{G}_{\text{user}}$ in (6) is formally defined as follows [35]:

$$u_i\left(p_i^*;p_{-i}^*\right) \ge u_i\left(p_i;p_{-i}^*\right), \forall\, p_i \in \mathcal{P}_i,\ \forall i \in \Omega. \tag{7}$$

It is known that, in a one-shot[7] noncooperative game, pure NEP is a critical operating point at which the outcome of the game becomes stabilized [35], and thus, it is of great interest to study the existence of NEP in such a game. Moreover, whether and how the noncooperative game can eventually arrive at the NEP is another question we have yet to answer. To this end, we first explicitly express the best response function of user i, i.e., $\mathcal{B}_i(p_{-i})$, which specifies the transmit power user i should use in response to the other users' power strategies and the price set by the relay. Specifically, depending on the price π_i set by the relay, the unique $\mathcal{B}_i(p_{-i})$ can be derived and expressed in a compact form as

$$\mathcal{B}_i(p_{-i}) = \left[\frac{\delta_i(\pi_i)\left(|h_i|^2 p_{\mathcal{R}} + N_0\right)\cdot\Delta_i}{|g_i|^2\cdot\left[|h_i|^2 p_{\mathcal{R}} - N_0\cdot\delta_i(\pi_i)\right]}\right]_0^{p_i^{\max}} \tag{8}$$

where $[\cdot]_a^b = \max\{\min\{\cdot, b\}, a\}$, $\Delta_i = \sum_{j=1,j\ne i}^{Q} |g_j|^2 p_j + N_0$, and $\delta_i(\pi_i)$ is a nonnegative and continuously nonincreasing function of π_i defined as

$$\delta_i(\pi_i) = \begin{cases} 0, & \text{if } \frac{1}{2} < \pi_i, \\ \frac{1}{2\pi_i} - 1, & \text{if } \left(1+\gamma_i\left(p_i^{\max};\mathbf{0}\right)\right)^{-1} < 2\pi_i \le 1, \\ \gamma_i\left(p_i^{\max};\mathbf{0}\right), & \text{if } 0 \le 2\pi_i \le \left(1+\gamma_i\left(p_i^{\max};\mathbf{0}\right)\right)^{-1}, \end{cases} \tag{9}$$

in which $\gamma_i(p_i^{\max};\mathbf{0})$ is obtained by plugging $(p_i;p_{-i}) = (p_i^{\max};\mathbf{0})$ into (2). The details of deriving (8) can be found in Appendix A. Denote $\mathbf{p} = (p_1, p_2 \cdots p_Q)$ and

[6]Note that the logarithm-based function or achievable rate is a widely-used utility definition (see, e.g., [9], [11], [17], [24], [25]) and the analysis herein can be applied, after modifications, to other concave utility functions as well. In particular, the existence of pure NEP and convergence of the iterative power control algorithm (developed in Section III-A) are not affected if we replace the achievable rate with a general concave utility function in (4). Furthermore, it is easy to incorporate a *weight* into the utility function, i.e., user i has a utility of $w_i R_i(p_i;p_{-i})$ where $w_i > 0$ is a factor that converts the achievable rate into currency [9] or approximates the reception quality in the case of video delivery applications [28].

[7]As will be shown later, the pure NEP is reached through an iterative power allocation process. Nevertheless, the user-level game in this paper is still one-shot in the sense that, unlike in a repeated game [35], the players or users do not take into account the history or future utility when making the current decisions. Thus, pure NEP is an appropriate concept characterizing the steady outcome of the game.

$\mathcal{B}(\mathbf{p}) = (\mathcal{B}_1(p_{-1}), \mathcal{B}_2(p_{-2}) \cdots \mathcal{B}_Q(p_{-Q}))$, respectively. Then, in order to facilitate the analysis and development of the distributed algorithm, we further simplify (8) and express it in a vector form as

$$\mathcal{B}(\mathbf{p}) = [\mathbf{T}\mathbf{p} + \mathbf{t_0}N_0]_{\mathbf{0}}^{\mathbf{p}^{\max}} \tag{10}$$

where $[\mathbf{a}]_{\mathbf{0}}^{\mathbf{p}^{\max}} = ([a_1]_0^{p_1^{\max}}, [a_2]_0^{p_2^{\max}} \cdots [a_Q]_0^{p_Q^{\max}})$, $\mathbf{t_0}_i = \frac{\delta(\pi_i)\cdot(|h_i|^2 p_{\mathcal{R}} + N_0)}{|g_i|^2 \cdot [|h_i|^2 p_{\mathcal{R}} - N_0 \cdot \delta(\pi_i)]}$, and

$$\mathbf{T}_{ij} = \begin{cases} \frac{\delta(\pi_i)\cdot\left(|h_i|^2 p_{\mathcal{R}} + N_0\right)}{|g_i|^2 \cdot [|h_i|^2 p_{\mathcal{R}} - N_0 \cdot \delta(\pi_i)]} \cdot |g_j|^2, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases} \tag{11}$$

for $i, j = 1, 2 \cdots Q$. Next, we present an iterative distributed algorithm (i.e., Algorithm I), in which each user chooses, at each iteration, its best response to the power strategies of the others.

Algorithm I: Iterative Distributed Power Allocation

Input: π_i, g_i, h_i **for user** i, $i = 1, 2 \cdots Q$

Step 1: $n = 0$; **choose any feasible** $\mathbf{p}^0 = (p_1^0, p_2^0 \cdots p_Q^0)$

Step 2: $\mathbf{p}^{(n+1)} = \mathcal{B}(\mathbf{p}^n)$

Step 3: $n = n + 1$; **go to Step 2 until convergence**

In Step 1 of Algorithm I, each user i can arbitrarily choose its initial power p_i^0 from its feasible power set $\{p_i : 0 \leq p_i \leq p_i^{\max}\}$ in a distributed manner and then, the initial power vector $\mathbf{p}^0 = (p_1^0, p_2^0 \cdots p_Q^0)$ is also feasible. To complete the algorithm description, we give Theorem 1 regarding the existence of NEP in the user game and the convergence of the proposed algorithm.

Theorem 1: Given any nonnegative price vector $\mathbf{\Pi} \succeq \mathbf{0}$ set by the relay, there exists a unique pure NEP of the user game $\mathcal{G}_{\text{user}}$. Moreover, starting from any initial point $\mathbf{p}^0 \in \mathcal{P} \triangleq \mathcal{P}_1 \times \mathcal{P}_2 \cdots \times \mathcal{P}_Q$, the iteration specified by $\mathbf{p}^{(n+1)} = \mathcal{B}(\mathbf{p}^n)$ always converges to the unique NEP of the user game as $n \to \infty$.

Proof: See Appendix B. □

Before concluding this section, we note that the distributed nature of the algorithm stems from the fact that the information required to compute $\mathcal{B}_i(p_{-i})$ at user i can be locally observed without exchanging CSI among different users. Specifically, as shown in (8), the information needed by user i includes the local CSI (i.e., g_i and h_i), the relay's transmit power $p_{\mathcal{R}}$, the price π_i set by the relay and the multiuser interference plus noise $\sum_{j=1,j\neq i}^{Q} |g_j|^2 p_j + N_0$. In particular, user i can obtain the local CSI through channel estimation and feedback.[8] The relay's transmit power $p_{\mathcal{R}}$ and the price π_i are transmitted via control channels to user i prior to the users' transmissions. Regarding the multiuser interference, the relay node can broadcast to all the users its amplification factor α such that user i, for $i \in \Omega$, acquires the value of $\sum_{j=1,j\neq i}^{Q} |g_j|^2 p_j + N_0$ by computing $\frac{p_{\mathcal{R}}}{\alpha^2} - |g_i|^2 p_i = \sum_{j=1,j\neq i}^{Q} |g_j|^2 p_j + N_0$. It can therefore be seen that the proposed algorithm can be applied in a distributed manner and that it needs to be re-executed when the price set by the relay is updated or the network condition changes. Finally, note that the proposed algorithm is applicable to scenarios in which the environment does not change frequently (e.g., the channel gains vary slowly when the nodes in the network move sufficiently slowly or remain in fixed positions). On the other hand, if the channels suffer from fast fading (e.g., due to high mobility), the proposed algorithm no longer works. The same limitation exists in (almost) all the existing work (see, for instance, [11], [16], [17], and [24]).

B. Uniform Pricing With Incomplete Information

In many wireless networks with limited information exchange among different nodes, the relay has only incomplete information about the users (e.g., the maximum power constraints of the users are private and thus unknown to the relay). Under such constraints, we propose a uniform pricing algorithm, i.e., the relay sets and broadcasts to the users a uniform price $\pi_1 = \pi_2 \cdots = \pi_Q = \pi$.

As we have stated in Theorem 1, the user-level game always consists of a unique NEP given any price vector set by the relay. Hence, the relay aims at maximizing its revenue by setting an appropriate price when the game reaches the NEP, i.e., the user game becomes stabilized. Nevertheless, since the private information of the users, e.g., power strategy space, is unknown to the relay, it cannot analytically compute the NEP of the user-level game $\mathcal{G}_{\text{user}}$ or directly set an optimal uniform price such that $\pi^* = \arg\max_{\pi \geq 0}(\pi \sum_i^Q \gamma_i(p_i^*; p_{-i}^*))$. As a consequence, an iterative process that adjusts the price is needed to identify the optimal uniform price. A naive solution is to perform brute-force exhaustive search. Specifically, the relay divides the range of feasible prices into many sufficiently small intervals, and for each small interval, the relay selects a uniform price that falls into the interval and computes the revenue when the user-level game reaches the unique NEP. Finally, the relay chooses the price that generates the maximum revenue among all the candidate prices. Unfortunately, the average total number of iterations required by this method to obtain the optimal uniform price is $m\bar{N}$, where m is the number of candidate prices and is typically a large value, and $\bar{N}$ is the average number of iterations needed by the distributed power allocation algorithm to converge.

Given that it is computationally prohibitive and mathematically involved to find the optimal uniform price through the exhaustive search, we alternatively propose a low-complexity algorithm that can yield a close-to-optimal uniform price. Before stating the algorithm, we first define the lower and upper bounds on the optimal uniform price, i.e., $\pi_a = \frac{1}{2}\min_{i\in\Omega}\{1 + \gamma_i(\mathbf{p}^{\max})\}^{-1}$ and $\pi_b = \frac{1}{2}\max_{i\in\Omega}\{1 + \gamma_i(\mathbf{p}^{\max})\}^{-1}$, respectively, and summarize some instrumental properties of the revenue function[9] $\rho(\pi) = \pi\sum_{i=1}^{Q}\gamma_i(\pi)$ in the following theorem.

Theorem 2: The revenue function has the following properties:

1) $\rho(\pi) \geq 0$;
2) $\rho(\pi) = 0$ if $\pi = 0$ or $\pi \geq \frac{1}{2}$;
3) $\rho(\pi) < \infty$ if the number of users, Q, is finite;

[8]At the beginning of a frame, a known pilot symbol is sent by a transmitter node to allow its receiver node to estimate the channel gain and then feed it back to the transmitter node. Other schemes are also available to let the users obtain their local CSI (see [32] and references therein for details).

[9]The SINR is an explicit function of the uniform price π which affects the net utility and the power allocation of users.

4) $\rho(\pi) = \pi \sum_{i=1}^{Q} \gamma_i(\mathbf{p}^{\max})$ when $0 \le \pi \le \pi_a$;
5) There exists a certain value of price $\hat{\pi}$ satisfying

$$\begin{cases} \hat{\pi} < \pi_b, & \exists i, j \in \Omega \; s.t. \; \gamma_i(\mathbf{p}^{\max}) \neq \gamma_j(\mathbf{p}^{\max}) \\ \hat{\pi} = \pi_b, & \forall \, i, j \in \Omega \; s.t. \; \gamma_i(\mathbf{p}^{\max}) = \gamma_j(\mathbf{p}^{\max}) \end{cases} \tag{12}$$

such that $\rho(\pi) = Q \cdot \left(\frac{1}{2} - \pi\right)$ if $\hat{\pi} \le \pi \le \frac{1}{2}$.

Proof: See Appendix C. ■

Theorem 2 can be interpreted as follows: $\langle 1 \rangle$ The receive SINR is always nonnegative and thus, the revenue is also nonnegative; $\langle 2 \rangle$ The revenue of the relay vanishes when the service of the relay, i.e., packet forwarding, is free or the price is too high; $\langle 3 \rangle$ The maximum revenue of the relay is finite as long as the number of users is finite; $\langle 4 \rangle$ and $\langle 5 \rangle$ The optimal price of the relay lies in a certain interval, i.e., $[\pi_a, \hat{\pi}]$, which depends on the channel conditions and transmit power constraints. Property 4 and 5 significantly reduce the complexity associated with the exhaustive search by eliminating the uniform prices that fall out of the range of the optimal price. They also form the basis of the proposed suboptimal uniform pricing algorithm. Specifically, the suboptimal uniform price is obtained by artificially shrinking the interval (i.e., $[\pi_a, \hat{\pi}]$) to a specific point which is then set as the uniform price. Following these desirable properties of the revenue function, we derive the following corollary.

Corollary 1: There exists an optimal finite uniform price π^* such that $\pi_a \le \pi^* \le \hat{\pi} \le \pi_b$, and the corresponding maximum revenue $\rho(\pi)$ is finite and positive. The equalities are activated simultaneously if and only if $\gamma_i(\mathbf{p}^{\max}) = \gamma_j(\mathbf{p}^{\max})$ for all $i, j \in \Omega$. ■

Corollary 1 states that the optimal price is upper and lower bounded by $\hat{\pi}$ and π_a, respectively. As a special case, if and only if $\gamma_i(\mathbf{p}^{\max}) = \gamma_j(\mathbf{p}^{\max})$ for all $i, j \in \Omega$, then we have $\pi_a = \pi^* = \hat{\pi} = \pi_b$. In other words, only when $\gamma_i(\mathbf{p}^{\max}) = \gamma_j(\mathbf{p}^{\max})$, for all $i, j \in \Omega$, the optimal uniform price can be analytically computed as $\pi_a = \pi^* = \hat{\pi} = \pi_b$. Otherwise, we only know $\pi_a \le \pi^* \le \hat{\pi} < \pi_b$, i.e., the optimal uniform price cannot be explicitly expressed in a closed form, although $\pi_a = \pi^* = \hat{\pi} < \pi_b$ may hold. If $\hat{\pi} = \pi_a$ holds, the optimal price π^* is then clearly π_a. Based on this fact, we propose a low-complexity algorithm that gives the relay a suboptimal price. Specifically, if we artificially increase π_a and decrease $\hat{\pi}$ simultaneously until they meet at $\overline{\pi}$ and assume that

$$\rho(\pi) = \begin{cases} \pi \cdot \sum_{i=1}^{Q} \gamma_i(\mathbf{p}^{\max}), & \text{if } 0 \le \pi \le \overline{\pi} \\ Q \cdot \left(\frac{1}{2} - \pi\right), & \text{if } \overline{\pi} < \pi \le \frac{1}{2} \end{cases} \tag{13}$$

we can easily obtain the "optimal" uniform price as

$$\pi^* \approx \overline{\pi} = \frac{Q}{2\left[\sum_{i=1}^{Q} \gamma_i(\mathbf{p}^{\max}) + Q\right]}. \tag{14}$$

Generally speaking, setting (14) as the price can only result in a suboptimal revenue for the relay. Nevertheless, the high computational complexity incurred by the exhaustive search is avoided and only limited information is needed to calculate (14): the number of active users in the network, i.e., Q, and the value of $\sum_{i=1}^{Q} \gamma_i(\mathbf{p}^{\max})$. The relay can set a sufficiently low[10] price π_s, given which the NEP is $\mathbf{p}^{\max}$, and find $\sum_{i=1}^{Q} \gamma_i(\mathbf{p}^{\max})$ by computing $\sum_{i=1}^{Q} \gamma_i(\mathbf{p}^{\max}) = \frac{\rho(\pi_s)}{\pi_s}$. The uniform price is determined in a similar way in the context of conventional cellular systems in [14] where the base station charges the users according to the throughput. Moreover, based on Corollary 1, we can establish the following corollary that guarantees the optimality of the uniform price in (14) when $Q = 1$.

Corollary 2: When there is only one user in the network, the uniform price in (14) is the optimal one that generates higher revenues than any other uniform prices, i.e.

$$\overline{\pi} = \frac{1}{2\left[\gamma_1\left(p_1^{\max}\right) + 1\right]} = \arg\max_{\pi \ge 0} \left(\pi \cdot \gamma_1\left(p_1^*\right)\right). \tag{15}$$

Furthermore, the transmit power of user 1 is $p_1^{\max}$ at the NEP of the game $\mathcal{G}_{\text{user}}$. ■

We note that Corollary 2 directly follows Corollary 1 by invoking $\pi_a = \pi^* = \hat{\pi} = \pi_b$ when $Q = 1$. Furthermore, when there are sufficiently many users in the network or the users operate in low SINR regions, (14) is also a good approximation of the optimal uniform price. Specifically, when the number of users in the network is large, the suboptimality of (14) can be explained as follows. It is natural that the level of interference observed by user i, i.e., $\sum_{j=1, j\neq i}^{Q} |g_j|^2 p_j$, increases when there are more active users. Hence, given a large value of Q, $\max_{i=1,2\cdots Q} \gamma_i(\mathbf{p}^{\max})$ becomes a small nonnegative number due to the strong interference caused by the other users. Correspondingly, the difference between the lower bound and the upper bound on the optimal uniform price is not significant, i.e., $\hat{\pi} - \pi_a$ is a small number. Thus, the suboptimal price (14), which lies between π_a and $\hat{\pi}$, is close to the optimal one. Note that the small nonnegative number $\hat{\pi} - \pi_a$ is also a upper bound on the gap between (14) and the optimal uniform price. Similar statements can be made when the network operates in low SINR regions as well. As in the existing literature (e.g., [14]), it is challenging to determine a priori the exact gap between (14) and the optimal uniform price, and hence, we shall verify in the section of numerical results that the loss of revenue is not significant in all the cases when the relay chooses (14), as compared to the optimal one obtained through exhaustive search, as its uniform price.

C. Differentiated Pricing With Complete Information

In this section, we extend the above analysis to a general case, in which different users may be charged at different prices, by considering that the relay has complete information about the network. It has been shown in [31] that the system performance can be improved if some users have complete information about the network. In the following analysis, the relay is assumed to know the maximum power constraints of all the users,[11] in addition to the channel coefficients. Under the differentiated pricing rule, we need to identify an optimal price vector $\mathbf{\Pi}^*$ set by the relay such that $\mathbf{\Pi}^* = \arg\max_{\mathbf{\Pi} \succeq \mathbf{0}} (\sum_{i=1}^{Q} \pi_i \gamma_i(p_i^*; p_{-i}^*))$. Differentiated pricing is also referred to as price discrimination in the economics literature [34]. Similarly, depending on the channel conditions and maximum power constraints, the relay can charge different users at different prices. Before developing

[10]It can be verified that, if the relay sets a price $0 < \pi \le \frac{1}{2} \min_{i \in \Omega} \left(1 + \gamma_i(p_i^{\max}; \mathbf{0})\right)^{-1}$, then the resulting NEP of the user game is $\mathbf{p}^{\max}$. The details are omitted due to the lack of space.

[11]To implement the protocol, the user may be required to report its maximum transmit power level to the relay before entering the network.

TABLE I
AVERAGE NUMBER OF ITERATIONS AND INFORMATION REQUIREMENT OF DIFFERENT PRICING ALGORITHMS

	Optimal Uniform	Sub-optimal Uniform	Differentiated
Information Requirement	$\varnothing$	$Q=\|\Omega\|$ and $\sum_{i=1}^{Q}\gamma_i(\mathbf{p}^{\max})$	g_i, h_i and $\mathcal{P}_i=\left\{p_i:0\le p_i\le p_i^{\max}\right\}$, for $i=1,2\cdots Q$
Average Number of Iterations	$m\bar{N}$	$\bar{N}$	0

the differentiated pricing algorithm, we first express the optimal value of π_i in terms of $\mathbf{p}^*$, for all $i\in\Omega$, in the following proposition.

Proposition 1: Assume that $\mathbf{\Pi}^*=\{\pi_1^*,\pi_2^*\cdots\pi_Q^*\}$ is the optimal price vector, which generates the maximum revenue for the relay, and that $\tilde{\mathbf{p}}^*=\{\tilde{p}_1^*,\tilde{p}_2^*\cdots\tilde{p}_Q^*\}$ is the unique corresponding power allocation vector at the NEP of the user game $\mathcal{G}_{\text{user}}$. Then, $\mathbf{\Pi}^*$ can be expressed in terms of $\tilde{\mathbf{p}}^*$ as follows:

$$\pi_i^*=\frac{1}{2\left(1+\gamma_i(\tilde{\mathbf{p}}^*)\right)},\quad \forall i\in\Omega \tag{16}$$

where $\gamma_i(\tilde{\mathbf{p}}^*)$ is obtained by substituting $\tilde{\mathbf{p}}^*$ into (2).

Proof: If $\mathbf{0}\prec\tilde{\mathbf{p}}^*\prec\mathbf{p}^{\max}$, then (16) directly follows the definition of NEP in (7) and the first-order optimality condition in (21). If $\tilde{p}_i^*=0$ for some $i\in\Omega_0\subset\Omega$, then $\pi_i^*\ge\frac{1}{2}$ and $\gamma_i(\tilde{\mathbf{p}}^*)=0$ and hence, (16) also holds true. If $\tilde{p}_i^*=p_i^{\max}$ for some $i\in\Omega_{\max}\subset\Omega$, then $\tilde{\mathbf{p}}^*$ is the power allocation vector at the NEP of the game $\mathcal{G}_{\text{user}}$ for any $0\le\pi_i\le\frac{1}{2(1+\gamma_i(\tilde{\mathbf{p}}^*))}$ and thus, it is clear that the optimal value of π is $\pi_i^*=\frac{1}{2(1+\gamma_i(\tilde{\mathbf{p}}^*))}$. Therefore, Proposition 1 is proved. ■

Proposition 1 enables us to express the price vector, which maximizes the relay's revenue, in terms of the transmit power levels at the NEP of the user-level game. Since the SINR is also a function of the transmit power levels of the users, we can then express the revenue, defined as the product of SINR and price, using a function of the power $\sum_{i=1}^{Q}\pi_i(\mathbf{p})\gamma_i(\mathbf{p})$. Therefore, instead of determining the optimal price vector directly, the relay can first decide the desired transmit power levels of the users and then set corresponding prices to enforce the users to transmit at these desired power levels. Mathematically, following Proposition 1 and substituting (16) into $\sum_{i=1}^{Q}\pi_i^*\gamma_i(p_i^*;p_{-i}^*)$, the problem of maximizing $\sum_{i=1}^{Q}\pi_i^*\gamma_i(p_i^*;p_{-i}^*)$ subject to $\mathbf{\Pi}^*\succeq\mathbf{0}$ can be reformulated as

$$\max\sum_{i=1}^{Q}\frac{\gamma_i(\tilde{\mathbf{p}}^*)}{2\left(1+\gamma_i(\tilde{\mathbf{p}}^*)\right)}=\frac{\sum_{i=1}^{Q}\frac{|h_i|^2p_{\mathcal{R}}}{|h_i|^2p_{\mathcal{R}}+N_0}|g_i|^2\tilde{p}_i^*}{2\left(\sum_{i=1}^{Q}|g_i|^2\tilde{p}_i^*+N_0\right)}$$
$$s.t.,\quad \mathbf{0}\preceq\tilde{\mathbf{p}}^*\preceq\mathbf{p}^{\max}, \tag{17}$$

where the objective function is linear-fractional and hence quasi-concave in $\tilde{\mathbf{p}}^*$ [36]. Therefore, the optimal value of $\tilde{\mathbf{p}}^*$ can be found by transforming (17) into a standard linear program [36], and the details of solving (17) are omitted due to the space limitations.

After the value of $\tilde{\mathbf{p}}^*$ is found, we can immediately obtain the optimal price vector using Proposition 1. It should be noted that, given the optimal price vector obtained using (16), the outcome of the game $\mathcal{G}_{\text{user}}$ when it reaches the NEP through iterations is that user i transmits at the power of $\tilde{p}_i^*$, regardless of the initial power strategies. This can be explained as follows. On one hand, we have shown in Proposition 1 that the optimal price vector $\mathbf{\Pi}^*$ can be expressed in (16) in terms of $\tilde{\mathbf{p}}^*$, i.e., one of the price vectors corresponding to $\tilde{\mathbf{p}}^*$ is $\mathbf{\Pi}^*$ given in (16). One the other hand, by uniqueness of the NEP of the game $\mathcal{G}_{\text{user}}$ given any price vectors stated in Theorem 2, it can be seen that $\tilde{\mathbf{p}}^*$ is the unique NEP of the game $\mathcal{G}_{\text{user}}$ if the relay sets $\mathbf{\Pi}^*$ as its pricing vector. Therefore, we can solve (17) to find $\tilde{\mathbf{p}}^*$ and then $\mathbf{\Pi}^*$ can be determined using (16). Furthermore, based on the objective function in (17), we have the following corollary regarding the upper bound on the revenue[12] of the relay.

Corollary 3: The maximum revenue that the relay can obtain from all the users by charging the optimal differentiated prices is upper bounded by $\frac{1}{2}$, and for any $i\in\Omega$, $\lim_{|g_i|^2,|h_i|^2\to\infty}\rho(\Pi^*)=\frac{1}{2}$. ■

Corollary 3 states that, given differentiated prices, the maximum revenue of the relay can be collected from only one user if this user has a sufficient good channel condition. In other words, to maximize its revenue with complete information, the relay can set an appropriate price vector such that only one user transmits, if this user's channel gains are sufficiently large (i.e., $|g_i|^2,|h_i|^2\to\infty$), while all the other users who are charged a price greater than or equal to $\frac{1}{2}$ remain silent. In contrast, under the uniform pricing algorithm, all the users are charged the same price according to (14) and hence, they will transmit simultaneously regardless of the channel conditions as long as the price is below $\frac{1}{2}$. Next, as a measure of comparison among different pricing schemes, we briefly discuss the average number of iterations played by the users and the information required by the relay to set the prices. Given complete information about the users, i.e., channel coefficients and power strategy space, the relay can directly compute the optimal differentiated price vector $\mathbf{\Pi}^*$, by solving the linear-fractional optimization problem in (17), and thus, it only needs to broadcast once the optimal price vector to the users. However, in the case of uniform pricing, the relay needs to set a sufficiently low price π_s before identifying the suboptimal uniform price, due to the constraint that only incomplete information about the users is available to the relay. Define $\bar{N}$ and m as the average number of iterations required by the user game $\mathcal{G}_{\text{user}}$ to converge and the number of candidate quantized values of uniform prices, respectively. We list in Table I the average number of iterations prior to data transmissions of the users, and the information requirement of different pricing schemes.

[12]The unit of revenue is the same as that of the utility function, i.e., "nats/s/Hz" in this paper [17]. Alternatively, the unit of the revenue can be converted to that of real money by multiplying the revenue with a constant converter without affecting the analysis [9].

D. System Utility Maximization

In the previous subsections, we have proposed two pricing mechanisms to maximize the relay's revenue, under the implicit assumption that the relay is solely revenue-driven. The proposed differentiated pricing algorithm, however, is also applicable if the relay wants to optimize the system utility which can be defined in any form. For the considered relay network, we have shown that, given any price vectors, there is a unique NEP in the user-level game, implying that the relay can set prices to enforce the users to transmit at desired power levels. Therefore, any system utility, defined as a function as the users' transmit power $\mathbf{p}$, can be achieved by setting appropriate prices.

As in [26], we denote the system utility which the relay wants to maximize as $U(\mathbf{p})$. Denote the optimal power levels maximizing $U(\mathbf{p})$ as $\bar{\mathbf{p}}$, i.e.

$$\bar{\mathbf{p}} = \arg\max_{\mathbf{p}\in\mathcal{P}} U(\mathbf{p}). \tag{18}$$

After finding[13] $\bar{\mathbf{p}}$, the relay can set prices according to

$$\pi_i = \frac{1}{2\left(1+\gamma_i(\bar{\mathbf{p}})\right)}, \quad \forall\, i \in \Omega. \tag{19}$$

Then, it is guaranteed that the users will transmit at $\bar{\mathbf{p}}$ at the NEP and thus, the system utility is maximized. For instance, let us take user scheduling as a concrete example. If the relay aims to schedule user 1 to transmit in a time slot and all the other users remain silent, it can set the price vector in such a way that $\pi_1 \in \left[0, \frac{1}{2}\right)$ and $\pi_2 = \cdots = \pi_Q \geq \frac{1}{2}$ and, given this price vector, only user 1 will transmit when the game reaches the NEP. We state the pricing-based utility maximization problem formally in the following proposition.

Proposition 2: Denote $\bar{\mathbf{p}} = \arg\max_{\mathbf{p}\in\mathcal{P}} U(\mathbf{p})$, where $U(\mathbf{p})$ is an arbitrary system utility. If the relay sets prices according to (19), then the system utility is maximized after the user-level game reaches the NEP.

Proof: By plugging $\pi_i = \frac{1}{2(1+\gamma_i(\bar{\mathbf{p}}))}$ and $p_{-i} = (\bar{p}_1 \cdots \bar{p}_{i-1}, \bar{p}_{i+1}, \cdots \bar{p}_Q)$ into the best response function of user i given in (8), it can be shown that

$$\mathcal{B}_i(p_{-i}) = \bar{p}_i, \tag{20}$$

for $i = 1, 2 \cdots Q$. Therefore, $\bar{\mathbf{p}}$ is the NEP of the user game if the prices are set according to (19). Then, by uniqueness of NEP in $\mathcal{G}_{\text{user}}$, we see that $\bar{\mathbf{p}}$ must be the transmit power levels at the NEP corresponding to the price vector set based on (19). As a result, the system utility is maximized. ■

Finally, we note that the relay is in fact taking the role of a *central planner* that has complete information about the network [26], if it wants to maximize the system utility which includes the revenue as a particular example. Nevertheless, the distinguishing feature of the proposed differentiated pricing algorithm is that it can enforce the users to transmit at desired power levels such that the system utility is maximized, even though these users are self-interested. Moreover, unlike in [26] wherein only near-optimal system utility can be achieved, we propose a pricing mechanism that can maximize any system utility by exploiting the uniqueness of NEP in the user-level game. The proposed pricing mechanism can be briefly described as follows. At the beginning of a frame, each user acquires its local information and, according to some performance metric (e.g., maximizing the revenue), the relay calculates the optimal power levels of all the users, sets its corresponding prices, and then announces the prices to the users. Then, Algorithm I is executed and the resulting NEP is achieved. In practice, Algorithm I can stop whenever the change in transmit power in two consecutive iterations is smaller than a certain threshold.

[13]If $U(\mathbf{p})$ is concave in $\mathbf{p}$, there exists efficient algorithms to maximize $U(\mathbf{p})$. Otherwise, the relay may need to maximize $U(\mathbf{p})$ via brutal-force search. As in [26], the details of optimizing $U(\mathbf{p})$ is beyond the scope of this paper, wherein we focus on the design of pricing algorithms.

V. Numerical Results

For the convenience of illustration, g_i and h_i are modeled as independently Rayleigh distributed random variables, for $i \in \Omega$. The transmit power of the relay node and the maximum transmit power of each source node are normalized to one.

We consider a simple four-user network and randomly generate the channel gains and illustrate in Fig. 1 the convergence of the proposed distributed power allocation algorithm and the suboptimal uniform pricing algorithm. The upper plot shows that the suboptimal price (dashed line) is reasonably close to the optimal price (solid line) obtained through exhaustive search, which validates the use of (14) as the uniform price selected by the relay. Next, we evaluate and compare the proposed pricing algorithms based on two distinct performance metrics, i.e., average[14] revenue and sum rate.[15] An orthogonal transmission scheme (i.e., time-division multiple access, or TDMA, in this paper) in which the source nodes do not interfere each other is also included in the comparison. Specifically, in the TDMA protocol, the users transmit in a round-robin manner and the relay charges each user using the optimal pricing scheme specified in (15).

A. Homogeneous Network Topology

Given the homogenous network topology, we assume that g_i and h_i have the same mean values, for $i \in \Omega$, i.e., $\mathbb{E}\{|g_1|^2\} = \mathbb{E}\{|h_1|^2\} = \cdots = \mathbb{E}\{|g_Q|^2\} = \mathbb{E}\{|h_Q|^2\}$, where $\mathbb{E}\{\cdot\}$ is the expectation operator.

1) Effects of Channel Gains: We consider a four-user network and examine the effects of channel gains on the average revenue and average sum rate in Fig. 2 and Fig. 3, respectively. As intuitively expected and can be seen from (17), the average revenue of the relay increases as the channel condition becomes better. Fig. 2 demonstrates that the revenue loss due to the suboptimality of the uniform price is not significant compared to the optimal uniform price. Among all the four pricing schemes, differentiated pricing generates the maximum revenue for the relay at the expense of having more information about the users. In other words, by allowing the users to transmit simultaneously, the differentiated pricing outperforms the optimal pricing in the TDMA protocol. This can be explained by noting that simultaneous transmission includes TDMA as a special case, i.e., simultaneous transmission reduces to TDMA if only one user is

[14]Throughout the simulations, "average" (e.g., average revenue, average rate) is taken over 10000 channel realizations.

[15]Due to the nonconvexity, we solve the problem of sum rate maximization in (18) using greedy methods and obtain (locally) optimal solutions.

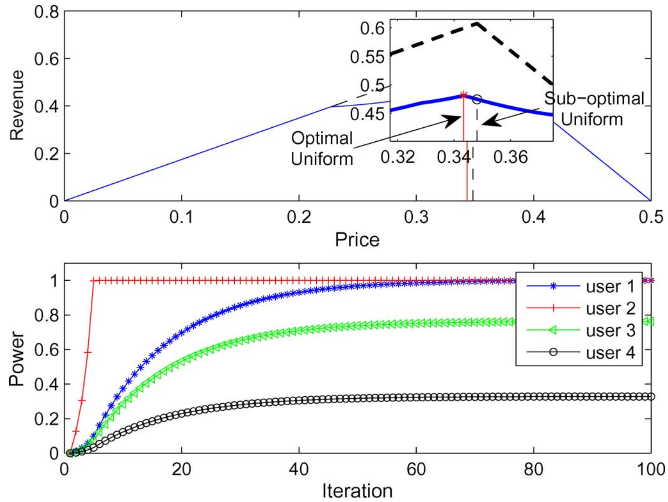

Fig. 1. Suboptimal pricing and convergence of distributed iterative power allocation algorithms.

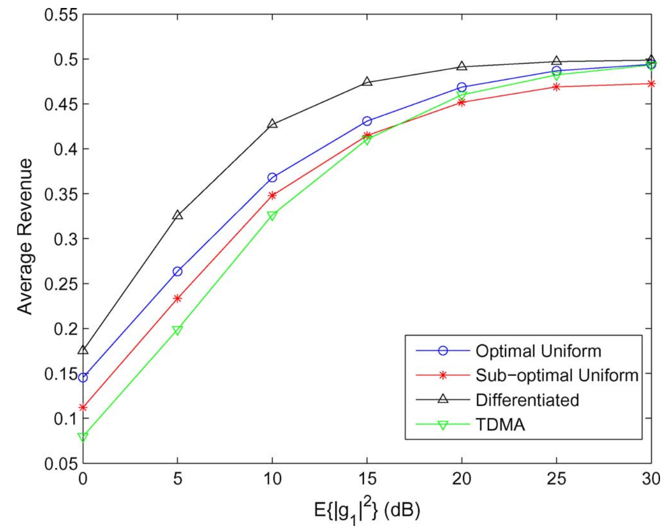

Fig. 2. Homogeneous network: average revenue versus average channel gain.

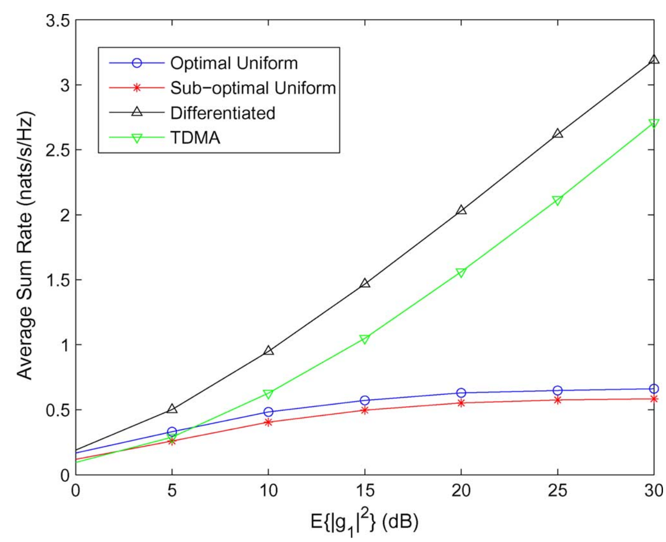

Fig. 3. Homogeneous network: average sum rate versus average channel gain.

scheduled to transmit at a time (the other users can be charged a price greater than or equal to $\frac{1}{2}$ such that they transmit at a zero power). Regarding the upper bound on the revenues, it can be observed that the maximum revenue is always less than $\frac{1}{2}$ regardless of the channel conditions, which verifies Corollary 3. Fig. 3 compares the the proposed algorithms when they are applied to maximize the average sum rate of all the users (i.e., the system utility function becomes the sum rate). Note that, although the suboptimal uniform pricing algorithm is applicable only for revenue maximization, we include the suboptimal pricing for the completeness of comparison when we consider sum rate maximization. The optimal uniform price is numerically searched such that the sum rate is maximized. The proposed differentiated pricing achieves the highest average sum rate among all the considered protocols, since it can enforce the selfish users to transmit at the optimal power through pricing. For instance, if a user has a poor channel condition, the relay can charge this user a price greater than or equal to $\frac{1}{2}$ such that this user keeps silent and does not cause interference to the other users. We also observe from Fig. 3 that, when the channel condition is good enough, the TDMA protocol outperforms the two uniform pricing schemes, in which all the source nodes always transmit simultaneously and the heavy interference among the source nodes significantly limits the achievable rate.

2) Effects of Number of Users: In Fig. 4 and Fig. 5, we fix the average channel gains and vary the number of active users. Fig. 4 shows that, when there are more users competing for the relay, the proposed pricing schemes achieve a higher revenue while the revenue obtained under the TDMA protocol does not change for the considered homogeneous network topology (since all the users with the same average channel statistics can be considered as one user in the TDMA protocol). Fig. 4 also indicates that the suboptimal revenue of the relay gained by setting (14) as the uniform price is close to the optimal uniform one obtained through exhaustive search. Like in Fig. 2, the differentiated pricing outperforms its uniform counterpart and the TDMA protocol in terms of the average revenue. In terms of the average sum rate, the differentiated pricing is still able to achieve the best performance, and the revenue under the two uniform pricing schemes decreases when there are more users simultaneously transmitting in the network due to the strong interference.

B. Heterogeneous Network Topology

For the convenience of illustration, we assume that $\mathbb{E}\{|g_1|^2\} = \mathbb{E}\{|h_1|^2\}$ and $\mathbb{E}\{|g_2|^2\} = \mathbb{E}\{|h_2|^2\} = \cdots = \mathbb{E}\{|g_Q|^2\} = \mathbb{E}\{|h_Q|^2\}$ in heterogenous network topologies.

1) Effects of Channel Gains: As an example, we focus on a four-user network with heterogenous channel conditions in Fig. 6 and Fig. 7. Fig. 6 demonstrates that, among all the four pricing schemes considered in this paper, the differentiated pricing yields the highest revenue for the relay, which is upper bounded by $\frac{1}{2}$. From Fig. 7, it can be seen that when the channel conditions becomes better, the average sum rate under the uniform pricing schemes are outperformed by that in the TDMA protocol and may not necessarily increase, since the interference also becomes stronger and reduces the received

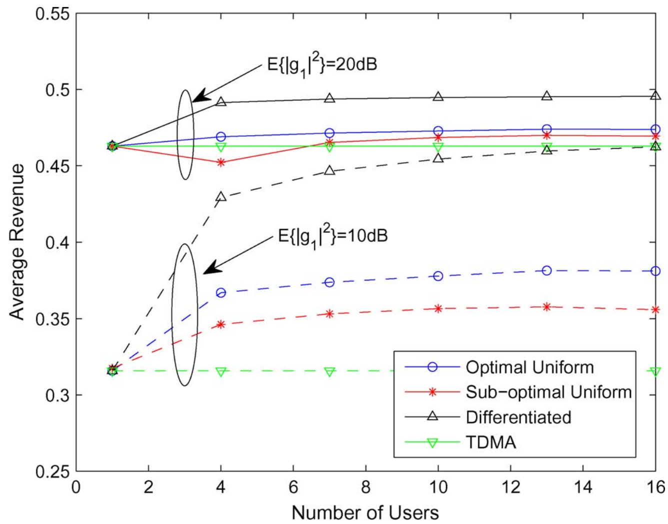

Fig. 4. Homogeneous network: average revenue versus number of users.

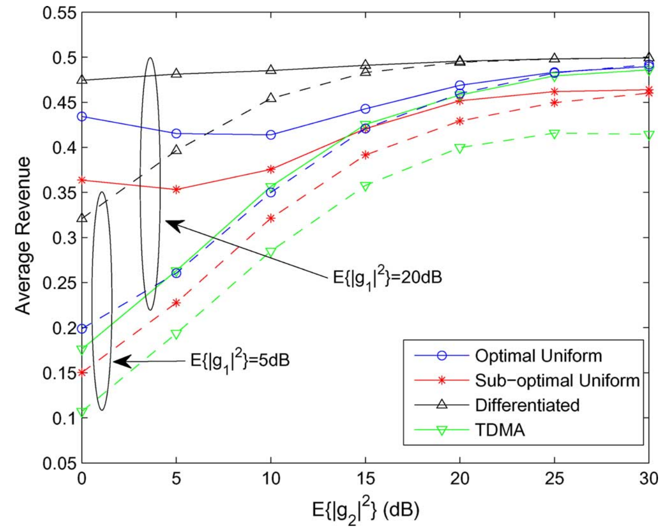

Fig. 6. Heterogeneous network: average revenue versus average channel gain.

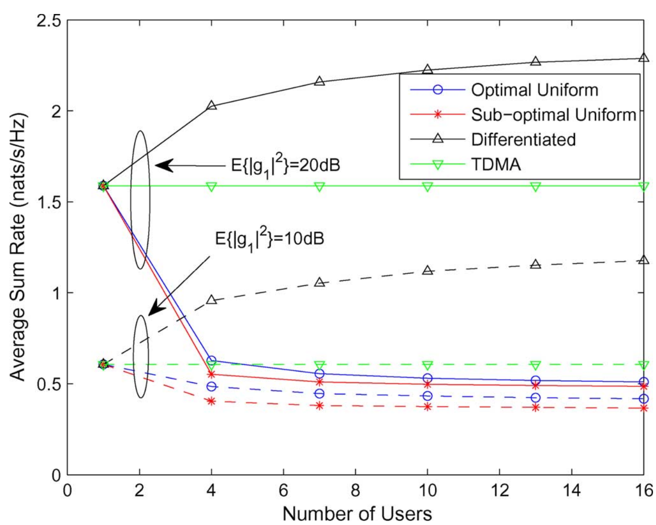

Fig. 5. Homogeneous network: average sum rate versus number of users.

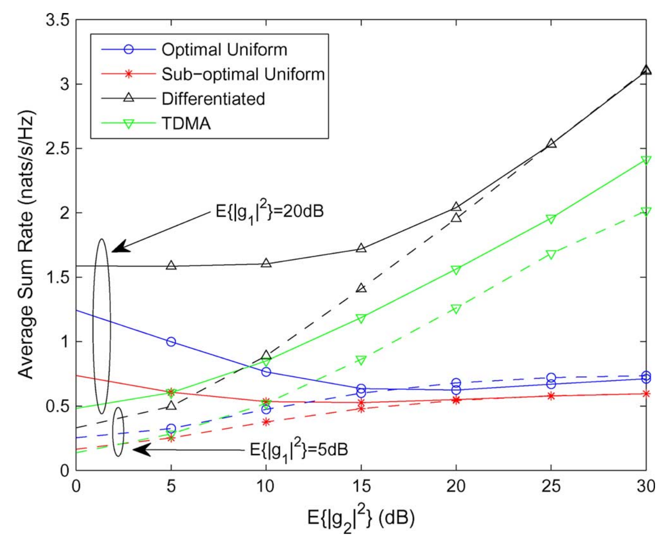

Fig. 7. Heterogeneous network: average sum rate versus average channel gain.

SINR. As in a homogeneous network topology, the proposed differentiated pricing achieves the highest average sum rate, since sum rate is only an instance of the system utility function and hence can be optimized using the differentiated pricing algorithm.

2) Effects of Number of Users: We consider fixed average channel gains and vary the number of users in a heterogeneous network topology in Fig. 8 and Fig. 9. It can be observed from Fig. 6 that the two uniform pricing schemes outperform the TDMA protocol in terms of the average revenue, and the proposed differentiated pricing achieves the highest revenue when there are more than one users regardless of the channel conditions. Fig. 9 shows that the differentiated pricing results in the highest average sum rate. Moreover, when the channel gains are strong, the average sum rate in the TDMA protocol is higher than that in the two uniform pricing schemes, since the strong interference can be avoided in the TDMA protocol.

To sum up, the proposed differentiate pricing scheme achieves the best performance in terms of the average revenue and sum rate, among all the four considered protocols. Compared to the TDMA protocol, the simultaneous transmission with uniform pricing schemes are generally more efficient in terms of the revenue, and less efficient in terms of the average sum rate (due to the unavoidable interference) when the channel gains are strong. Prior to concluding this section, we note that the analysis of general system utility maximization via the differentiated pricing in Section IV-D is also valid and can be applied to arbitrary utility functions, though we do not show it in the simulations due to space limitations.

VI. Conclusion

In this paper, we considered a wireless relay network consisting of one relay node and multiple source-destination pairs. First, the interactions between the relay and the users were appropriately captured. We then modeled each user as a self-interested player, which aims at maximizing its own benefit by choosing the optimal transmit power, and analyzed the competition among the users using the notion of noncooperative

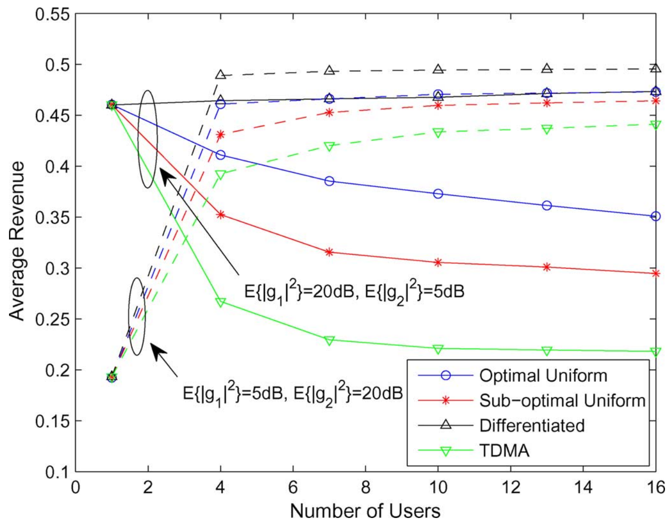

Fig. 8. Heterogeneous network: average revenue versus number of users.

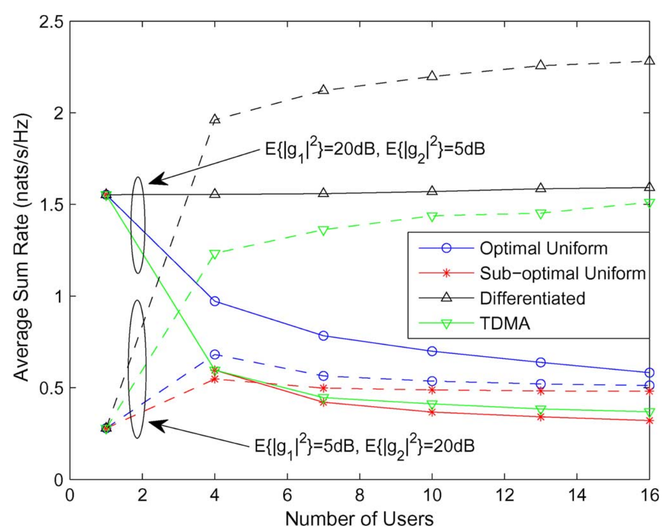

Fig. 9. Heterogeneous network: average sum rate versus number of users.

game theory. It was proved that, in the noncooperative game played by the users, there always exists a unique steady operating point, i.e., NEP, which can be achieved in a distributed manner. Next, under the assumption that the relay has only incomplete information about the users, we proposed a low-complexity algorithm, in which the relay charges the users at a suboptimal uniform price. The analysis was then extended to differentiated pricing wherein the relay charges different users at different prices. We also showed that the proposed differentiated pricing can be applied to maximize any system utility. Extensive simulations showed that the relay can gain the maximum revenue and the maximum sum rate by adopting the differentiated pricing algorithm, which requires complete information about the users. Furthermore, given only incomplete information about the users, the relay can apply the proposed low-complexity suboptimal uniform pricing algorithm without incurring a significant revenue loss as compared to the optimal uniform pricing algorithm. Interference cannot be avoided when using uniform pricing schemes and thus, the resulting average sum rate is less than that achieved by orthogonal transmission (e.g., TDMA) when the channels are in a good condition.

Appendix A

We first express the first-order partial derivative of u_i with respect to p_i as

$$\frac{\partial u_i(p_i;p_{-i})}{\partial p_i} = \left(\frac{1}{2(1+\gamma_i)} - \pi_i\right) \cdot \frac{\partial \gamma_i}{\partial p_i}, \tag{21}$$

where γ_i is given in (2) and

$$\frac{\partial \gamma_i(p_i;p_{-i})}{\partial p_i} = \frac{\Delta_i \cdot \left(|h_i|^2 p_{\mathcal{R}} + N_0\right) \cdot |g_i|^2 |h_i|^2 p_{\mathcal{R}}}{\left[|g_i|^2 N_0 p_i + \left(|h_i|^2 p_{\mathcal{R}} + N_0\right) \cdot \Delta_i\right]^2} \tag{22}$$

is clearly always positive, in which $\Delta_i = \sum_{j=1, j\neq i}^{Q} |g_j|^2 p_j + N_0$. It can then be shown that, given any particular value of p_{-i}, we have $0 \leq \gamma_i(p_i;p_{-i}) \leq \gamma_i(p_i^{\max};p_{-i})$. Depending on price π_i set by the relay, we need to consider the following four cases to find the best response of user i.

Case 1) $\frac{1}{2} < \pi_i$. It is clear that, in this case, $\frac{1}{2(1+\gamma_i)} - \pi_i$ is negative for any feasible value of $p_i \in [0, p_i^{\max}]$. Thus, $\frac{\partial u_i(p_i;p_{-i})}{\partial p_i}$ is negative and the net utility function $u_i(p_i;p_{-i})$ is monotonically decreasing in $p_i \in [0, p_i^{\max}]$, following which we see that the best response of user i should be transmitting at a zero power.

Case 2) $\frac{1}{2[1+\gamma_i(p_i^{\max};p_{-i})]} \leq \pi_i \leq \frac{1}{2}$. There exists a unique $p_i^* \in \mathcal{P}_i$ such that $\frac{\partial u_i(p_i^*;p_{-i})}{\partial p_i} = 0$, and $p_i^* = \left[\frac{\left(\frac{1}{2\pi_i}-1\right)\left(|h_i|^2 p_{\mathcal{R}}+N_0\right)\left(\sum_{j=1,j\neq i}^{Q}|g_j|^2 p_j+N_0\right)}{|g_i|^2 \cdot \left[|h_i|^2 p_{\mathcal{R}} - N_0 \cdot \left(\frac{1}{2\pi_i}-1\right)\right]}\right]_0^{p_i^{\max}}$. Moreover, it follows that $\frac{\partial u_i(p_i;p_{-i})}{\partial p_i} > 0$ if $0 \leq p_i \leq p_i^*$, and $\frac{\partial u_i(p_i;p_{-i})}{\partial p_i} > 0$ if $p_i^* < p_i^* \leq p_i^{\max}$. Therefore, p_i^* is the unique best response of user i.

Case 3) $\frac{1}{2[1+\gamma_i(p_i^{\max};\mathbf{0})]} < \pi_i < \frac{1}{2[1+\gamma_i(p_i^{\max};p_{-i})]}$. In this case, we have $\frac{1}{2(1+\gamma_i)} - \pi_i \geq \frac{1}{2[1+\gamma_i(p_i^{\max};p_{-i})]} - \pi_i > 0$. Thus, the net utility function of user i is increasing in $p_i \in [0, p_i^{\max}]$ and the best response of user i is $p_i^{\max}$.

Case 4) $0 \leq \pi_i < \frac{1}{2[1+\gamma_i(p_i^{\max};\mathbf{0})]}$. As in Case 3, it can be shown that the best response of user i is $p_i^{\max}$.

By jointly considering all the possible four cases and after some simple mathematical manipulations, we can express the best response function of user i in a compact form given in (8).■

Appendix B

The proof is mainly based on the *standard* interference function that was first proposed for distributed power control in [21]. Any function $f(\mathbf{x})$ satisfying the following three properties, for all $\mathbf{x} \geq \mathbf{0}$, is called *standard*:

1) Positivity: $f(\mathbf{x}) > 0$;
2) Monotonicity: if $\mathbf{x} \geq \tilde{\mathbf{x}}$, then $f(\mathbf{x}) \geq f(\tilde{\mathbf{x}})$;
3) Scalability: for all $\beta > 1$, $\beta f(\mathbf{x}) > f(\beta \mathbf{x})$.

To prove the existence of a unique NEP and the convergence of Algorithm I, we consider the following two cases depending on the value of $\mathbf{\Pi}$ which plays a critical role in the best response vector $\mathcal{B}(\mathbf{p})$.

Case 1: $\mathbf{0} \preceq \mathbf{\Pi} \prec \frac{\mathbf{1}}{\mathbf{2}}$.

We have in this case $0 < \mathbf{t}_{\mathbf{0}_i} < +\infty$ and $0 < \mathbf{T}_{ij} < +\infty$, for $i \neq j$ and $i, j = 1, 2 \cdots Q$. It is trivial to show that, without considering the maximum power constraint, the function of $\mathbf{p}$ in the update (10), i.e., $\mathbf{T}\mathbf{p} + \mathbf{t_0} N_0$, is standard. Then, following the proof of Theorem 7 in [21], we can easily prove that the update rule defined in (10) with the maximum power constraint is also standard. Hence, by applying Corollary 1 in [21], we establish the existence of a unique fixed point in the proposed iterative power allocation process (i.e., NEP of the user game) and the convergence of Algorithm I to this unique NEP, given $\mathbf{0} \preceq \mathbf{\Pi} \prec \frac{\mathbf{1}}{\mathbf{2}}$.

Case 2: $\frac{1}{2} \leq \pi_i$, for some $i \in \Omega$.

In this case, the iteration $p_i^{n+1} = \mathcal{B}_i(\mathbf{p}^n)$ is always zero, if $\frac{1}{2} \leq \pi_i$. Thus, users that are charged a price greater than or equal to $\frac{1}{2}$ can be excluded from the network. The remaining users are all charged with a price less than $\frac{1}{2}$ and hence, they form a new *virtual* network that satisfies *Case 1*. Hence, as we have shown in *Case 1*, the game played by the users in the virtual network admits a unique NEP that can be reached by applying Algorithm I. Note that adding the users that are charged a price greater than or equal to $\frac{1}{2}$ into the virtual network has no effect for the virtual network, since the added users always transmit at zero powers. Therefore, the game has a unique NEP and the proposed distributed power allocation algorithm converges to this unique NEP regardless of the initial point, even though $\mathcal{B}(\mathbf{p})$ is not a standard interference function as it violates the properties of *positivity* and *scalability*.

To sum up, we have proved Theorem 1 by considering the above two cases. The existence of a NEP can also be proved by showing that the net utility function of each user is quasi-concave in this user's power and continuous in the power of all the users, and that the feasible power set is compact and closed. The details are omitted for brevity. It should also be noted that, in general, the existence of a (even unique) fixed point of an iterative process does not necessarily imply the convergence of this iterative process (see [33] for an example). The existence of a fixed point and convergence are two separate properties of an iterative process. In the problem considered in this paper, however, both the existence of a fixed point (NEP) and the convergence of the iterative process can be established, since the best response function is standard and there exists a maximum power constraint [21]. ∎

APPENDIX C

The proof is given in the order of the properties listed in Theorem 3.

Property 1–3 directly follows the best response function in (8).

Property 4: Given $p_{-i} = (p_1^{\max} \cdots p_{i-1}^{\max}, p_{i+1}^{\max} \cdots p_Q^{\max})$, it can be derived from the best response function that $\mathcal{B}_i(p_{-i}) = p_i^{\max}$, if $0 \leq \pi \leq \frac{1}{2}\{1 + \gamma_i(\mathbf{p}^{\max})\}^{-1}$. Hence, $\mathbf{p}^{\max} = (p_1^{\max}, p_2^{\max} \cdots p_Q^{\max})$ satisfies $\mathbf{p}^{\max} = \mathcal{B}(\mathbf{p}^{\max})$, i.e., $\mathbf{p}^{\max}$ is the NEP, when $0 \leq \pi \leq \frac{1}{2}\min_{i=1,2\cdots Q}\{1 + \gamma_i(\mathbf{p}^{\max})\}^{-1}$. In this case, by Theorem 2, the distributed power allocation algorithm globally converges to the unique point $\mathbf{p}^{\max}$. The intuitive interpretation is that, when the price is sufficiently low, every user can afford the payment charged by the relay and thus will transmit at a high power. When all the users transmit at their maximum powers, the receive SINR $\gamma_i(\pi)$ is a positive constant, denoted by $\tilde{\gamma}_i = \gamma_i(\mathbf{p}^{\max})$, for $i = 1, 2 \cdots Q$, irrespective the value of π. Therefore, the revenue $\rho(\pi) = \pi \sum_j^Q \tilde{\gamma}_j$ is a strictly increasing function of π when $0 \leq \pi \leq \frac{1}{2}\min_{i=1,2\cdots Q}\{1 + \gamma_i(\mathbf{p}^{\max})\}^{-1}$.

Property 5: We first introduce the following lemma before proving the existence of $\hat{\pi}$.

Lemma 1: If $\pi > \frac{1}{2}\{1 + \gamma_i(\mathbf{p}^{\max})\}^{-1}$, then the maximum transmit power constraint of user i is not activated at the NEP of the game $\mathcal{G}_{\text{user}}$, i.e., $0 \leq p_i^* < p_i^{\max}$, for any $i = 1, 2 \cdots Q$.

Proof: By taking the first-order derivative of $\mathcal{B}_i(p_{-i})$ in (8) with respect to π, it can be easily shown that $\mathcal{B}_i(p_{-i})$ is a strictly decreasing function of π when $\frac{1}{2}\{1 + \gamma_i(p_i^{\max}; p_{-i})\}^{-1} \leq \pi \leq \frac{1}{2}$. In particular, $\pi = \frac{1}{2}\{1 + \gamma_i(p_i^{\max}; p_{-i})\}^{-1}$ results in $\mathcal{B}_i(p_{-i}) = p_i^{\max}$. Therefore, the maximum power constraint of user i is not activated, i.e., $0 \leq \mathcal{B}_i(p_{-i}) < p_i^{\max}$, if $\frac{1}{2}\{1 + \gamma_i(\mathbf{p}^{\max})\}^{-1} < \pi \leq \frac{1}{2}$.

When $\pi > \frac{1}{2}$, $\mathcal{B}_i(p_{-i})$ is always zero. Hence, Lemma 1 is proved. □

Now, we shall prove Property 5 by considering the following two cases.

Case 1: $\gamma_i(\mathbf{p}^{\max}) = \gamma_j(\mathbf{p}^{\max})$, for $i, j = 1, 2 \cdots Q$.

In this case, we will show that $\hat{\pi} = \frac{1}{2}\max_{i=1,2\cdots Q}\{1 + \gamma_i(\mathbf{p}^{\max})\}^{-1}$. When $\hat{\pi} \leq \pi \leq \frac{1}{2}$, $\mathcal{B}_i(p_{-i})$ satisfies $\frac{\partial u_i(p_i; p_{-i})}{\partial p_i}|_{p_i = \mathcal{B}_i(p_{-i})} = 0$, for $i = 1, 2 \cdots Q$. Then, it can be derived that

$$\gamma_i\left[\mathcal{B}_i(p_{-i}); p_{-i}\right] = \frac{1}{2\pi} - 1, \tag{23}$$

for any $p_{-i} \in \mathcal{P}_1 \times \cdots \mathcal{P}_{i-1} \times \mathcal{P}_{i+1} \cdots \mathcal{P}_Q$. Thus, at the NEP of the game $\mathcal{G}_{\text{user}}$, we have $\gamma_i(p_i^*; p_{-i}^*) = \frac{1}{2\pi} - 1$, for $i = 1, 2 \cdots Q$. Therefore, the revenue at the relay, i.e., $\rho(\pi) = \pi \sum_{i=1}^Q \gamma_i(p_i^*; p_{-i}^*) = Q \cdot \left(\frac{1}{2} - \pi\right)$, is a strictly decreasing function of π when $\hat{\pi} = \frac{1}{2}\max_{i=1,2\cdots Q}\{1 + \gamma_i(\mathbf{p}^{\max})\}^{-1}$.

Case 2: "$\gamma_i(\mathbf{p}^{\max}) = \gamma_j(\mathbf{p}^{\max})$, for $i, j = 1, 2 \cdots Q$" does not hold.

Without loss of generality, we assume $\{1 + \gamma_1(\mathbf{p}^{\max})\}^{-1} = \max_{i=1,2\cdots Q}\{1 + \gamma_i(\mathbf{p}^{\max})\}^{-1}$ and $\{1 + \gamma_Q(\mathbf{p}^{\max})\}^{-1} = \min_{i=1,2\cdots Q}\{1 + \gamma_i(\mathbf{p}^{\max})\}^{-1}$. Lemma 1 states that, for any value of the price $\pi > \frac{1}{2}\{1 + \gamma_Q(\mathbf{p}^{\max})\}^{-1}$, the maximum transmit power constraint of user Q is not activated at the NEP, i.e., $p_Q^* < p_Q^{\max}$. Then, following the proof of Lemma 1, it can be also shown that $p_i^* < p_i^{\max}$, for $i = 1, 2 \cdots Q - 1$, if $\pi \geq \frac{1}{2}\{1 + \gamma_1(\mathbf{p}^{\max})\}^{-1}$.

By temporarily relaxing the maximum power constraint, we can express the best response function in (10) as

$$\mathbf{p}^{(n+1)} = \mathcal{B}(\mathbf{p}^n) = \mathbf{T}\mathbf{p}^n + \mathbf{t_0} N_0, \tag{24}$$

where $\mathbf{T}$ is defined in (11). It was shown in [22] that, if and only if the spectral radius of $\mathbf{T}$ is less than one, the iteration process specified by (24) converges to a unique fixed point, regardless of the initial point, and the fixed point is given by

$$\mathbf{p}^* = \mathbf{T}\mathbf{p}^* + \mathbf{t_0}N_0 = (\mathbf{I}-\mathbf{T})^{-1}\mathbf{t_0}N_0 = \sum_{i=0}^{\infty}\mathbf{T}^i\mathbf{t_0}N_0. \quad (25)$$

As stated above, when $\pi \geq \frac{1}{2}\{1+\gamma_1(\mathbf{p}^{\max})\}^{-1}$, the transmit power of user i is less than its maximum power constraint, for $i = 1, 2\cdots Q$, and hence, we have

$$\begin{aligned}\mathbf{p}^* &= [\mathbf{T}\mathbf{p}^* + \mathbf{t_0}N_0]_{\mathbf{0}}^{\mathbf{p}^{\max}} = \mathbf{T}\mathbf{p}^* + \mathbf{t_0}N_0 \\ &= (\mathbf{I}-\mathbf{T})^{-1}\mathbf{t_0}N_0 = \sum_{i=0}^{\infty}(\mathbf{T})^i\mathbf{t_0}N_0. \quad (26)\end{aligned}$$

It should be noted that, if (26) holds, $\gamma_i(p_i^*; p_{-i}^*) = \frac{1}{2\pi} - 1$ can be satisfied at the NEP of the user game, for $i = 1, 2\cdots Q$, and as a consequence, the revenue of the relay, i.e., $\rho(\pi) = \pi\sum_{i=1}^{Q}\gamma_i(p_i^*; p_{-i}^*) = Q\cdot\left(\frac{1}{2}-\pi\right)$, is a strictly decreasing function of π.

As each nondiagonal element of $\mathbf{T}$ is continuously decreasing in π, it is clear that the transmit power of each user at the NEP, given in terms of the fixed point in (25), is also decreasing in π, if the NEP in the game $\mathcal{G}_{\text{user}}$ without considering the maximum power constraint exists. Thus, the minimum price, denoted by $\hat{\pi}$, which yields a matrix $\mathbf{T}$ with a spectral radius of less than one and satisfies (26) must be less than $\frac{1}{2}\{1+\gamma_1(\mathbf{p}^{\max})\}^{-1}$ and, given the minimum price, only one user reaches its maximum power constraint or multiple (less than Q) users reach their corresponding maximum power constraints simultaneously at the NEP. On the other hand, if $\hat{\pi}$ is less than or equal to $\frac{1}{2}\{1+\gamma_Q(\mathbf{p}^{\max})\}^{-1}$, at least one user will violate the maximum power constraint at the fixed point of the iteration process specified by (24) and hence, (26) cannot be satisfied. Therefore, $\hat{\pi}$ lies between $\frac{1}{2}\{1+\gamma_Q(\mathbf{p}^{\max})\}^{-1}$ and $\frac{1}{2}\{1+\gamma_1(\mathbf{p}^{\max})\}^{-1}$, and when $\hat{\pi} \leq \pi \leq \frac{1}{2}$, the revenue at the relay is a strictly decreasing function of π.

By considering Case 1 and Case 2 separately, we have proved Property 5. This proves Theorem 3. ■

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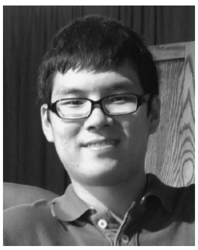

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