

Game Theoretic Design of MAC Protocols: Pricing and Intervention in Slotted–Aloha

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Abstract—In many wireless communication networks a common channel is shared by multiple users who must compete to gain access to it. The operation of the network by self-interested and strategic users usually leads to the overuse of the channel resources and to substantial inefficiencies. Hence, incentive schemes are needed to overcome the inefficiencies of non-cooperative equilibrium. In this work we consider a slotted-Aloha random access protocol and two incentive schemes: *pricing* and *intervention*. We provide some criteria for the designer of the protocol to choose one scheme between them and to design the best policy for the selected scheme, depending on the system parameters. Our results show that intervention can achieve the maximum efficiency in the *perfect monitoring* scenario. In the *imperfect monitoring* scenario, instead, there exists a threshold for the number of users such that, for a number of users lower than the threshold, intervention outperforms pricing, whereas, for a number of users higher than the threshold pricing outperforms intervention.

Index Terms—MAC protocols, Slotted–Aloha, Game Theory, Incentive schemes, Pricing, Intervention, Imperfect monitoring

I. INTRODUCTION

We consider a slotted-Aloha random access protocol, where each user transmits within a slot according to some user-chosen probability. Without any further mechanism, self-interested users would implement the *always transmit* strategy, resulting in the network collapse. To make the network robust to selfish users, it is fundamental to design a scheme that provides to the users the incentives to adopt a better (from the network designer’s point of view) strategy.

In the past decade a lot of research was devoted to the development of such incentive schemes for slotted-Aloha random access protocols. Some of this research, such as [1]–[5], adopts pricing schemes that charge the users for their resource usage.¹ In this way, it is in the self-interest of each user to limit its access probability.

Recently, a new incentive scheme, called *intervention*, has been proposed in [6] and has been applied to MAC problems in [7] and [8]. In this scheme, an *intervention device* is placed in the network. Such a device can monitor the users’ behavior and intervene affecting the users’ resource usage. The action of the intervention device depends on the actions of the users. The intervention device provides the incentives for the

users to obey a given access probability rule by threatening *punishments* if users disobey.

In this paper we provide the tools to design pricing and intervention schemes to make a random access protocol robust against strategic users. As in most of the previous works in pricing and intervention, we consider only *linear intervention* and *linear pricing* schemes, because they are simple to implement and yet efficient enough to achieve high performance (or even optimality in some cases). Simple rules are important in particular for pricing schemes, because the users might not accept to pay for their resource usage following complex rules.

The complexity of the design process and the performance achievable depend on various features of the system, such as the number of users, the users’ heterogeneity, and the capability of monitoring the users’ actions. To the best of our knowledge, this is the first work that compares intervention and pricing in terms of the network environment. We focus on a simple MAC protocol, slotted-Aloha, because it makes it possible to formulate a simple game in which the outcomes can be computed analytically, to highlight the consequence of not taking into account the strategic nature of some users when designing a MAC protocol, and to obtain important insights about possible solutions to such a problem. For these features, slotted-Aloha is widely used in game theoretic studies [1]–[5], [7], [8]. The extension of this paper to more realistic MAC protocols will be considered in future works.

This paper is divided into two main parts. In the first part, we consider the *perfect monitoring* scenario, i.e., we assume that the users’ actions are estimated without errors. We show that intervention can achieve the maximum efficiency, i.e., the maximum social welfare, while pricing is able to reach an efficient use of the network resources but the positive payments subtracted from the users’ utilities prevent it to achieve the maximum social welfare. In the second part, we consider an *imperfect monitoring* scenario, assuming that a uniformly distributed noise term is added to the estimated actions. In the imperfect monitoring scenario, the performance of the intervention scheme degrades considerably as the number of users increases, and there exists a threshold for the number of users such that, for a number of users lower than the threshold, the intervention scheme outperforms the pricing scheme, whereas for a number of users higher than the threshold the pricing scheme outperforms the intervention scheme. The analysis in this paper can serve as a guideline for a designer to select between pricing and intervention and to design the best policy for the selected scheme, depending on some system parameters

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¹In the literature pricing schemes may refer also to distributed schemes in which the users are cooperative and *fictional* prices are used to obtain an efficient distributed algorithm. Here we consider strategic and selfish users; thus, to be effective, the pricing scheme requires the users to pay real money.

such as the number of users and the intensity of the monitoring noise.

The remainder of this paper is organized as follows. In Section II we describe the considered MAC protocol. We introduce the games that model the interaction between strategic users and we formulate the problem of designing efficient incentive schemes in Section III. In Section IV we derive the optimal pricing and intervention schemes to adopt in the perfect monitoring scenario and we quantify the performance achievable. We consider the imperfect monitoring scenario in Section V. Section VI concludes with some remarks.

II. SYSTEM MODEL

We consider a wireless network of n users that share a common channel and we make the following assumptions for the contention model:

- Time is slotted and slots are synchronized;
- Users always have packets to transmit in every slot;
- If a packet is received, the receiver immediately sends an acknowledgment (ACK) packet;
- The transmission of a packet and the (possible) corresponding ACK are completed within a slot;
- A packet is received successfully if and only if it does not collide with other transmissions;
- Each user i selects a transmission probability $p_i \in [0, 1]$ at the beginning of the communication and will transmit with the same probability p_i in every time slot, i.e., there are no adjustments in the transmission probabilities. This excludes coordination among users, for example, using time division multiplexing.

Notice that ACK packets are always successfully received because they are transmitted over idle channels.

Denoting with $p = (p_1, \dots, p_n)$ the transmission probability vector, the average throughput (in packets per slot) of user i is given by

$$T_i(p) = p_i \prod_{j=1, j \neq i}^n (1 - p_j) \quad (1)$$

The resource usage of user i is therefore proportional to i 's transmission probability.

We assume that the utility of user i is given by

$$U_i(p) = \theta_i \ln T_i(p) \quad (2)$$

where the parameter $\theta_i > 0$ allows to differentiate between different classes of users. The higher θ_i , the higher user i 's valuation of the throughput. The logarithm makes the utility a concave function, which models the fact that the users usually have more desire to increase their own throughput when it is low than when it is high.

We define the social welfare of the network as the sum of all users' utilities:

$$U(p) = \sum_{i=1}^n U_i(p) \quad (3)$$

Finally, the network is said to operate optimally if the users choose the transmission probabilities that maximize Eq. (3). It is straightforward to check that the Hessian of $U(p)$ is a diagonal matrix with strictly negative diagonal entries, therefore it

is negative definite. Imposing the partial derivatives equal to 0, the unique transmission probability vector $p^* = (p_1^*, \dots, p_n^*)$ that maximizes Eq. (3) is given by

$$p_k^* = \frac{\theta_k}{\sum_{i=1}^n \theta_i} \quad , \quad k = 1, \dots, n \quad (4)$$

We say that $U(p^*)$ is the maximum efficiency utility.

In order to adopt the optimal transmission probability, the users need to know the sum of the valuations θ_i of the other users. This information must be spread in the network at the beginning of the communication. This can be done either in a distributed way or in a centralized way. In particular, in the last case an entity (e.g., a predetermined user or the access point) might collect the users' valuations and broadcast to all users the value $\sum_{i=1}^n \theta_i$. Once the users have this information, they can locally compute their optimal transmission probabilities according to Eq. (4) and adopt them.

III. GAME MODEL AND DESIGN PROBLEM

While the network optimal transmission policy p^* is easy to compute, the actual transmission probability selected by each user depends on the objective of that user. If the users are compliant with the optimal policy, then they compute and adopt p^* and the network operates optimally. However, if the users are self-interested and strategic, instead of complying with the optimal policy they will adopt the transmission probabilities that optimize their own utility. Since the interests of individual users are different from the interests of the group of users as a whole, the network might (and usually will) operate inefficiently.

To analyze the interaction between strategic decision-makers, we exploit the models offered by game theory [9]. We define the contention game

$$\Gamma = (\mathcal{N}, A, \{U_i(\cdot)\}_{i=1}^n) \quad (5)$$

where $\mathcal{N} = \{1, 2, \dots, n\}$ denotes the set of users, $A = \times_{i=1}^n [0, 1]^n$ denotes the action space and $U_i : A \rightarrow \mathbb{R}$ is the utility of a generic user i , defined by Eq. (2). The action for user i represents the transmission probability p_i chosen by user i . Throughout the paper, we will use the terms action and transmission probability interchangeably, and similarly for action profile (a collection of the users' actions) and transmission probability vector.

A widely accepted solution concept for non-cooperative games is the Nash Equilibrium (NE), defined as the action profile p^{NE} so that each user obtains its maximum utility given the actions of the other users, i.e.,

$$U_i(p^{NE}) \geq U_i(p_i, p_{-i}^{NE}) \quad , \quad \forall i \in \mathcal{N} \quad , \quad \forall p_i \in [0, 1]$$

where p_{-i} denotes the transmission probabilities of all the users except for user i .

An action profile p is a NE of the contention game Γ if and only if at least one user adopts a transmission probability equal to 1. Moreover, $p_i = 1$ is a *weakly dominant action* for every user i , i.e., $u_i(1, p_{-i}) \geq u_i(p)$, for every action profile p . Therefore, in our contention game, each user has an incentive to adopt the *always transmit* strategy, resulting in network collapse.

Here we ask if it is possible to design the network to make it robust against strategic users. We want to introduce some mechanism to deter the users from adopting high transmission probabilities. The incentive schemes we consider belong to two classes:

- Pricing: users are charged depending on their transmission probabilities
- Intervention: the users' resource usage is affected by the *intervention device*, in a way that depends on the users' transmission probabilities

The interaction between the designer, the users and the system can be roughly summarized into three stages, (1) the design stage, (2) the information exchange stage, and (3) the transmission stage.

In the *design stage* the designer designs the pricing or intervention scheme. Specifically, the designer predicts strategic users' actions given any pricing or intervention scheme, and chooses the pricing or intervention scheme that results in the most desired outcome. This is done once, then the designer leaves the system forever. Notice that, to efficiently design these schemes, the designer has to know how pricing or intervention affect the users' utilities.

In the *information exchange stage* some useful information is collect and, possibly, distributed. The intervention device (or the device that manages the payments in the pricing scheme) has to identify the users that are connected to the network, has to inform them about the adopted intervention or pricing scheme, and has to estimate the action they select, e.g., by counting the number of correct transmissions of each user in a certain time interval. To consider the impact of an imperfect estimation we will denote by \hat{p}_i the estimated action of user i , by \hat{p} the estimated action profile and by $\pi_i(\hat{p}_i | p_i)$ the probability density function of i 's estimated action, given that i 's action is p_i . We say that the monitoring is *perfect* if the users' actions are estimated without errors, i.e., \hat{p}_i coincides with p_i (in this case $\pi_i(\hat{p}_i | p_i)$ might be thought as a Dirac delta function centered in p_i). We say that the monitoring is *imperfect* if the estimates are affected by errors, i.e., there is a positive probability that \hat{p}_i is different from p_i .

In the *transmission stage* the users transmit packets adopting their constant transmission probabilities and, in the meantime, they have to pay for their resource usage based on the pricing scheme, or their resource usage is affected based on the intervention scheme.

In this paper we play the role of a benevolent designer that seeks to design the pricing and intervention rules to maximize the social welfare of the system in the transmission stage. We neglect the social welfare obtained in the information exchange stage because we assume that the transmission stage is much longer than that of the information exchange stage.

A. Pricing

Pricing schemes use monetary charges to deter users' greediness. If i 's payment is increasing in i 's resource usage, user i might find it convenient to limit its transmission probability. In general, user i is charged according to the *pricing rule* $f_i^P : [0, 1] \rightarrow \mathfrak{R}$, which is a function of i 's estimated action

\hat{p}_i . Assuming that the payments affect additively the users' utilities, i 's expected utility is given by

$$U_i^P(p) = \theta_i \ln T_i(p) - \int_0^1 \pi_i(\hat{p}_i | p_i) f_i^P(\hat{p}_i) \partial \hat{p}_i \quad (6)$$

Once a pricing scheme is selected and communicated to the users, the interaction among users can be modeled through the game

$$\Gamma^P = \left(\mathcal{N}, A, \{U_i^P(\cdot)\}_{i=1}^n \right) \quad (7)$$

Among all the possible pricing rules, there is one class of rules that is particularly interesting, namely, the class of *linear pricing rules*, in which users are charged linearly with respect to their transmission probabilities, i.e.,

$$f_i^P(\hat{p}_i) = c_i \hat{p}_i \quad (8)$$

where $c_i \geq 0$ is the unit price. We restrict our attention to the linear pricing rules, as done in most of the pricing literature, because they are computationally simple to implement and we do not lose much, in term of performance, in doing so.

Once the prices $c = (c_1, \dots, c_n)$ are fixed, since we will prove the existence and uniqueness of the *NE* of the game Γ^P , the social welfare can be uniquely determined. The goal of the designer is to choose the unit prices $c = (c_1, \dots, c_n)$ to maximize the social welfare, i.e., it has to solve the following Pricing Design (**PD**) problem:

$$\begin{aligned} \mathbf{PD} \quad & \underset{c}{\operatorname{argmax}} \sum_{i \in \mathcal{N}} U_i^P(p^{NE}) \\ & \text{subject to:} \\ & c_i \geq 0, \quad \forall i \in \mathcal{N} \\ & U_i^P(p^{NE}) \geq U_i^P(p_i, p_i^{NE}), \quad \forall p_i \in [0, 1], \quad \forall i \in \mathcal{N} \end{aligned}$$

B. Intervention

In the intervention framework the designer deploys in the network an intervention device that monitors the users' actions and can intervene adopting itself an action that affects the users' resource usage. In our case, we assume that the intervention device is able to correctly recognize the packets transmitted by different users and to estimate the users' actions. If the packet of a generic user i is correctly received, the intervention device may choose to jam its ACK² depending on the estimate of its action. Specifically, the intervention device jams the ACK sent to user i with a probability that is given by the *intervention rule* $f_i^I : [0, 1] \rightarrow [0, 1]$, which is a function of the estimated action \hat{p}_i .

The *intervention level* $f_i^I(\hat{p}_i)$ must be interpreted as a *punishment* to user i after having deviated from a recommended (socially-beneficial) action. Such punishments are a threat to users, and must be designed such that the users find in their self-interest to adopt the recommended actions. At the same time, when users adopt the recommended actions, the intervention level must be minimized (possibly, nullified), to avoid to decrease the users' utilities.

²Many works on security, such as [10]–[12], take into consideration the possibility of performing intelligent jamming in which the jamming signal is concentrated on control packets.

Different from pricing, intervention changes the structure of the utility of each user, affecting directly their resource usage. In fact, the average throughput of user i is now given by

$$T_i^I(p) = p_i \left(1 - \int_0^1 \pi_i(\hat{p}_i | p_i) f_i^I(\hat{p}_i) \partial \hat{p}_i \right) \prod_{j=1, j \neq i}^n (1-p_j) \quad (9)$$

where $\int_0^1 \pi_i(\hat{p}_i | p_i) f_i^I(\hat{p}_i) \partial \hat{p}_i$ represents the *average intervention level*.

The utility of user i is modified accordingly

$$U_i^I(p) = \theta_i \ln T_i^I(p) \quad (10)$$

Once the intervention rules are selected and communicated to the users, the interaction between the users can be modeled through the game

$$\Gamma^I = \left(\mathcal{N}, A, \{U_i^I(\cdot)\}_{i=1}^n \right) \quad (11)$$

Among all the possible intervention rules, there is one class of rules that is particularly interesting, namely, the class of *affine intervention rules*. $f_i^I : [0, 1] \rightarrow [0, 1]$ is an affine intervention rule if

$$f_i^I(\hat{p}_i) = [r_i(\hat{p}_i - \tilde{p}_i)]_0^1 \quad (12)$$

for certain parameters $\tilde{p}_i \in [0, 1]$ and $r_i \geq 0$, where $[\cdot]_a^b = \min\{\max\{a, \cdot\}, b\}$.

In an affine intervention rule, \tilde{p}_i represents a target action for user i , while r_i is the rate of increase of the intervention level due to an increase in i 's action. If the estimated action \hat{p}_i is lower than or equal to the target action \tilde{p}_i , then the intervention level is equal to 0. If \hat{p}_i is higher than \tilde{p}_i , then the intervention level is proportional to $\hat{p}_i - \tilde{p}_i$, until it saturates to 1.

For $r_i \rightarrow +\infty$, the intervention device jams the ACKs sent to user i whenever it detects that i is adopting an action higher than the target one. Such a rule, which we refer to as an *extreme rule*, represents the strongest punishment that the intervention device can adopt.

We restrict our attention to the affine intervention rules because they are computationally simple to implement and we do not lose much, in term of performance, in doing so (as we will see, in some cases such rules are even able to achieve the benchmark optimum).

Once the parameters $\tilde{p} = (\tilde{p}_1, \dots, \tilde{p}_n)$ and $r = (r_1, \dots, r_n)$ are fixed, and assuming that the users coordinate to the best (from the social welfare point of view) *NE* of the game Γ^I ³, the social welfare can be determined. The goal of the designer is to choose the parameters \tilde{p} and r to maximize the social welfare, i.e., it has to solve the following Intervention Design (**ID**) problem:

$$\begin{aligned} \mathbf{ID} \quad & \operatorname{argmax}_{\tilde{p}, r} \left[\max_{p^{NE}} \sum_{i \in \mathcal{N}} U_i^I(p^{NE}) \right] \\ & \text{subject to:} \\ & \tilde{p}_i \in [0, 1], \quad r_i \geq 0, \quad \forall i \in \mathcal{N} \\ & U_i^I(p^{NE}) \geq U_i^I(p_i, p_{-i}^{NE}), \quad \forall p_i \in [0, 1], \quad \forall i \in \mathcal{N} \end{aligned}$$

³The existence of *NEs* will be proved and it is easy to coordinate the users to the best *NE*. In fact, we will prove that the socially best *NE* is also the best for each user, and it is uniquely determined by \tilde{p} .

Differently from the **PD** problem, the **ID** problem requires a maximization with respect to the *NEs* because of the non uniqueness of the *NE*.

IV. PERFECT MONITORING

In this section we assume that the estimated actions are equal to the real actions, i.e., $\hat{p}_i = p_i$, for every user $i \in \mathcal{N}$. Hence, in Eq. (6) and (9) the integrals must be substituted, respectively, with $f_i^P(p_i)$ and $f_i^I(p_i)$. In the following we compute the optimal linear pricing scheme and affine intervention rule that a designer should adopt to maximize the social welfare if the monitoring is perfect.

A. Pricing design

Given a linear pricing scheme c_i , $i \in \mathcal{N}$, the interaction between users in the perfect monitoring scenario adopting pricing is modeled with the game

$$\Gamma^P = \left(\mathcal{N}, A, \{U_i^P(\cdot)\}_{i=1}^n \right) \quad (13)$$

where

$$U_i^P(p) = \theta_i \ln \left[p_i \prod_{j=1, j \neq i}^n (1-p_j) \right] - c_i p_i \quad (14)$$

The goal of the designer is to design the unit prices c to maximize the social welfare in the presence of strategic users, solving the **PD** problem with the utilities given by Eq. (14).

Lemma 1. *The unique NE of the game Γ^P is $p_k^{NE} = \frac{\theta_k}{c_k}$, $k \in \mathcal{N}$.*

Proof: To compute the best response function of users k , we use the first order condition. First, we check that $U_k^P(p)$ is concave in p_k (i.e., the second derivative with respect to p_k is negative). Then, we set to 0 the first derivative of $U_k^P(p)$, with respect to p_k .

$$\begin{aligned} \frac{\partial U_k^P(p)}{\partial p_k} &= \frac{\theta_k}{p_k} - c_k, \quad \frac{\partial^2 U_k^P(p)}{\partial p_k^2} = -\frac{\theta_k}{p_k^2} < 0 \\ \frac{\partial U_k^P(p)}{\partial p_k} = 0 &\rightarrow p_k = \frac{\theta_k}{c_k} \end{aligned}$$

Proposition 1. *The optimal pricing scheme to adopt is $c_k^* = \sum_i \theta_i$.*

Proof: We want to find the unit prices c_k , $k \in \mathcal{N}$, so that the social welfare $U(p) = \sum_{i=1}^n U_i^P(p)$ is maximized, assuming that the users adopt the *NE* action profile (i.e., we have to substitute c_k with $\frac{\theta_k}{p_k}$ into the expression of $U(p)$). We first prove that $U(p)$ is a (multivariable) concave function, by checking its Hessian.

$$\begin{aligned} \frac{\partial U(p)}{\partial p_k} &= \frac{\theta_k}{p_k} - \frac{\sum_{i \neq k} \theta_i}{1-p_k} \\ \frac{\partial^2 U(p)}{\partial p_k^2} &= -\frac{\theta_k}{p_k^2} - \frac{\sum_{i \neq k} \theta_i}{(1-p_k)^2} < 0, \quad \frac{\partial^2 U(p)}{\partial p_k \partial p_i} = 0 \quad \forall i \neq k \end{aligned}$$

The Hessian of $U(p)$ is negative definite (it is a diagonal matrix with strictly negative diagonal entries), so $U(p)$ is

concave. Thus, the global maximizer of $U(p)$ can be obtained with the first order condition: $\frac{\partial U(p)}{\partial p_k} = 0 \rightarrow p_k = \frac{\theta_k}{\sum_i \theta_i} \rightarrow c_k = \sum_i \theta_i, \forall k \in \mathcal{N}$. ■

Notice that the transmission probabilities adopted by the users in the optimal pricing policy are equal to the transmission probabilities adopted by compliant users to maximize the social welfare, i.e., $p_k^{NE} = \frac{\theta_k}{c_k} = p^*$, where p^* is defined in Eq. (4).

B. Intervention design

Given an affine intervention rule r_i and $\tilde{p}_i, i \in \mathcal{N}$, the interaction between users in the perfect monitoring scenario adopting intervention is modeled with the game

$$\Gamma^I = \left(\mathcal{N}, A, \{U_i^I(\cdot)\}_{i=1}^n \right) \quad (15)$$

where

$$U_i^I(p) = \theta_i \ln \left[p_i \left(1 - [r_i(p_i - \tilde{p}_i)]_0^1 \right) \prod_{j=1, j \neq i}^n (1 - p_j) \right] \quad (16)$$

The goal of the designer is to design the intervention rule to maximize the social welfare in the presence of strategic users, solving the **ID** problem with the utilities given by Eq. (16).

To maximize the social welfare the designer has to guarantee a positive throughput to every user. For this reason, in the following we focus on intervention rules in which $\tilde{p}_k \in (0, 1), \forall k$. In fact, if $\tilde{p}_k = 1$ the intervention device never jams the ACK sent to user k , consequently $p_k = 1$ represents a weakly dominant action and the throughput of all the users except k is 0. If $\tilde{p}_k = 0$ user k is punished whenever it transmits with positive probability, and it is possible to show that in this case there always exists a non-zero \tilde{p}_k such that the utility for user k is higher than the one obtainable with $\tilde{p}_k = 0$, with no impact on the utilities of the other users.

In the intervention framework an action profile p in which at least two users transmit with probability 1 always represents a *NE*, regardless of the particular intervention rule adopted. We refer to such equilibria as *trivial NEs*. They correspond to the worst (socially and individually) case possible, in which the throughput of all users is equal to 0. However, by accurately choosing the parameters r_i and $\tilde{p}_i, i \in \mathcal{N}$, it is possible to coordinate the users to a *NE* in which they all obtain a positive throughput.

Lemma 2. For any $\tilde{p} = (\tilde{p}_1, \dots, \tilde{p}_n)$, if $r_k \geq \frac{1}{\tilde{p}_k}$ for every user $k \in \mathcal{N}$, then \tilde{p} is the unique non trivial *NE* of the game Γ^I .

Proof: We can write $r_k = \frac{1}{\tilde{p}_k + \delta}$, for some constant $\delta > -\tilde{p}_k$. Then, if $p_k < \tilde{p}_k$,

$$U_k^I(p_k, \tilde{p}_{-k}) = \theta_k \ln \left[p_k \prod_{j \neq k} (1 - \tilde{p}_j) \right]$$

If $\tilde{p}_k \leq p_k \leq 2\tilde{p}_k + \delta$,

$$U_k^I(p_k, \tilde{p}_{-k}) = \theta_k \ln \left[\frac{-p_k^2 + 2\tilde{p}_k p_k + \delta p_k}{\tilde{p}_k + \delta} \prod_{j \neq k} (1 - \tilde{p}_j) \right]$$

If we exclude the trivial *NEs* from the analysis, we can limit the search of user k 's best response in $[0, 2\tilde{p}_k + \delta]$, because if user k selects a higher transmission probability the intervention level is equal to 1, resulting in a throughput equal to 0 for user k . The derivative of k 's utility, with respect to k 's action, is equal to

$$\frac{\partial U_k^I}{\partial p_k} = \begin{cases} \frac{\theta_k}{p_k} & \text{if } p_k < \tilde{p}_k \\ \theta_k \frac{2(\tilde{p}_k - p_k) + \delta}{p_k(2\tilde{p}_k - p_k + \delta)} & \text{if } \tilde{p}_k \leq p_k \leq 2\tilde{p}_k + \delta \end{cases}$$

If $\delta \leq 0$ (i.e., $r_k \geq \frac{1}{\tilde{p}_k}$), $U_k^I(p_k, \tilde{p}_{-k})$ is continuous, increasing in p_k for $p_k < \tilde{p}_k$, and decreasing otherwise. Thus, \tilde{p}_k is the best action for user k . If $\delta > 0$ (i.e., $r_k < \frac{1}{\tilde{p}_k}$), $U_k^I(p_k, \tilde{p}_{-k})$ is continuous, increasing in p_k for $p_k < \tilde{p}_k + \frac{\delta}{2}$, and decreasing otherwise. Thus, $\tilde{p}_k + \frac{\delta}{2}$ is the best action for user k . The two analyzed cases imply that \tilde{p} is a *NE* if and only if $r_k \geq \frac{1}{\tilde{p}_k}, \forall k$. ■

The proof of Lemma 2 shows that \tilde{p}_k is a weakly dominant action, for every user k , and that \tilde{p} is individually and socially better than any trivial *NE*. Hence, though it is not the unique equilibrium, it is expected that the users coordinate to \tilde{p} .

Proposition 2. The optimal affine intervention rule to adopt is $r_k \geq \frac{1}{p_k^*}$ and $\tilde{p}_k = p_k^*$, for every user k , where p_k^* is defined in Eq. (4).

Proof: Given the actions of the users, the utility of a user and the social welfare are decreasing as the intervention level for that user increases. However, using the intervention rule $r_k \geq \frac{1}{p_k^*}$ and $\tilde{p}_k = p_k^*, \forall k$, the users have the incentive to adopt the action profile $p = p^*$ and, at the same time, the intervention level they are subjected to is equal to 0. Thus, the outcome of the system is equal to the benchmark optimum. Finally, this implies that $r_k \geq \frac{1}{p_k^*}$ and $\tilde{p}_k = p_k^*$ define an optimal affine intervention rule, and, in fact, also an optimal intervention rule within the class of all intervention rules. ■

Corollary 1. The optimal affine intervention rule is optimal in the class of all intervention rules.

C. Comparison between pricing and intervention

In the perfect monitoring scenario, by adopting either pricing or intervention the designer can provide the incentive for strategic users to choose the optimal action profile of Eq. (4). Hence, the efficiency of the utilization of the channel resource is optimized with respect to the valuations $\theta_i, i \in \mathcal{N}$, of the users. However, there is a big difference between pricing and intervention. Intervention schemes reach this objective by threatening the users to intervene if they do not follow the recommendations, although at the equilibrium the intervention is not triggered and therefore the resource usage is not affected. Conversely, pricing schemes charge each user that transmits with a positive probability, thus affecting its utility and the social welfare. Hence, only the intervention scheme is able to achieve the optimal social welfare that can be obtained when users behave cooperatively, i.e., when they comply to a prescribed protocol that maximizes the social welfare.

In Fig. 1 the social welfare and the total throughput in the perfect monitoring scenario are plotted as a function of the number of users in the system. We consider the case in which the users behave cooperatively, and the cases in which the users' actions are enforced by the pricing and intervention schemes derived in Sections IV-A and IV-B. A symmetric scenario is considered, i.e., $\theta_i = 1, \forall i \in \mathcal{N}$. Thus, the optimal transmission policy in the cooperative case, defined by Eq. (4), is $p_k^* = \frac{1}{n}$, for every user k .

The results confirm the above discussion: both schemes are able to obtain the same total throughput of the cooperative case, but only the intervention scheme is able to maximize the (total) users' satisfaction. In fact, there is a finite gap, which increases as the number of users increases, between the optimal social welfare and the one achievable with the pricing scheme. Finally, notice that the social welfare always decreases as the number of users increases because there are more collisions and the number of unexploited slots increases, resulting in an inefficient utilization of the channel; this is an unavoidable consequence of the lack of coordination.

V. IMPERFECT MONITORING

We now study whether the qualitative results obtained for the perfect monitoring scenario still hold for the imperfect monitoring case. In this section we will see that there is a substantial difference for the intervention scheme when the monitoring is imperfect. The intuition behind it is related to the possibility that the estimation errors trigger the intervention even though the users are adopting the recommended actions. For the pricing scheme, if the expectations of the estimated actions are equal to the real actions, each user might be overcharged or undercharged but, on average, it is charged correctly, therefore the performance is not strongly affected.

The imperfect monitoring model we consider for the estimation of user i 's action is an additive noise term that is uniformly distributed in $[-\epsilon_i, \epsilon_i]$, with $0 < \epsilon_i \ll 1$, i.e.,

$$\hat{p}_i = [p_i + n_i]_0^1, \quad n_i \sim \mathcal{U}[-\epsilon_i, \epsilon_i] \quad (17)$$

In the following we compute the best linear pricing scheme and affine intervention rule that a designer should adopt to maximize the social welfare, assuming that both the designer and the users are aware of the estimation errors and know their distribution (17). The interaction among users must be

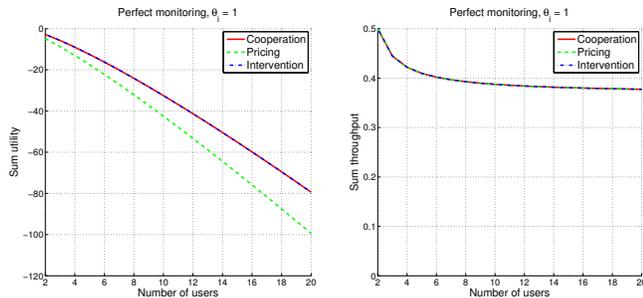


Fig. 1. Social welfare and total throughput vs. number of users, in the perfect monitoring scenario

modeled through the games (7) and (11) and the designer has to solve the **PD** and **ID** problems using the utilities given by Eq. (6) and (10). With a similar approach, it is possible to analyze the situations in which 1) neither the designer nor the users are aware of the estimation errors, and 2) only the designer is aware of the estimation errors. Due to space constraints, we do not report the analysis of these two cases in this paper, the interested reader can find it in [13].

A. Pricing design

Once the pricing scheme is given, the interaction among users can be modeled with the game in Eq. (7), where

$$U_i^P(p) = \theta_i \ln \left[p_i \prod_{j=1, j \neq i}^n (1 - p_j) \right] - \frac{c_i}{2\epsilon_i} \int_{-\epsilon_i}^{\epsilon_i} [p_i + x]_0^1 \partial x$$

Denote

$$\mathcal{C}(\epsilon) = \left\{ x : \frac{1}{2} \leq x \leq 1 - \epsilon \text{ and } x \ln x - x \geq \frac{\epsilon}{4} - 1 \right\}$$

$$\bar{p}_k = \begin{cases} -\frac{\epsilon_k}{2} + \frac{1}{2} \sqrt{\epsilon_k^2 + \frac{8\epsilon_k \theta_k}{c_k}} & \text{if } \frac{\theta_k}{c_k} < \epsilon_k \\ \frac{\theta_k}{c_k} & \text{if } \epsilon_k \leq p_k \leq \frac{1}{2} \text{ or } p_k \in \mathcal{C}(\epsilon_k) \\ 1 & \text{otherwise} \end{cases}$$

Lemma 3. \bar{p}_k is the unique NE of the game Γ^P .

Proof: Due to space constraints, we only provide a sketch of the proof. We refer the interested reader to [13] for the complete proof. The second derivative of $U_i(p)$, with respect to p_i , must be computed. It results that $U_i(p)$, with respect to p_i , is concave in $[0, \max(\sqrt{\frac{2\epsilon_i \theta_i}{c_i}}, 1 - \epsilon_i)]$, and convex otherwise. If $\frac{1}{2} < \frac{\theta_i}{c_i} \leq 1 - \epsilon_i$, after the change of concavity $U_i(p)$ decreases to a local minimum and then increases again. In this case, user i 's best action is obtained comparing the two local maxima. Otherwise, user i 's best action is the unique local maximum. ■

Consider the following notation:

$$p_{k,4} = \frac{\theta_k(2 - \epsilon_k)}{4 \sum_i \theta_i} + \frac{1}{2} \sqrt{\left[\frac{\theta_k(2 - \epsilon_k)}{2 \sum_i \theta_i} \right]^2 + 4 \frac{\theta_k \epsilon_k}{2 \sum_i \theta_i}}$$

$$p_{k,5} = \max \mathcal{C}(\epsilon_k)$$

Proposition 3. The optimal unit price c_k to adopt is, $\forall k \in \mathcal{N}$,

$$c_k = \begin{cases} \frac{2\epsilon_k \theta_k}{p_{k,4}(p_{k,4} + \epsilon_k)} & \text{if } p_{k,4} < \epsilon_k \\ \frac{\theta_k}{\epsilon_k} & \text{if } p_{k,4} \geq \epsilon_k \text{ and } \frac{\theta_k}{\sum_i \theta_i} \leq \epsilon_k \\ \sum_i \theta_i & \text{if } \epsilon_k \leq \frac{\theta_k}{\sum_i \theta_i} \leq \frac{1}{2} \text{ or } \frac{\theta_k}{\sum_i \theta_i} \in \mathcal{C}(\epsilon_k) \\ \frac{\theta_k}{p_{k,5}} & \text{otherwise} \end{cases}$$

Proof: Due to space constraints, we only provide a sketch of the proof. We refer the interested reader to [13] for the complete proof. Considering that users adopt the unique NE

\bar{p}_k , it is possible to compute the social welfare $U(p)$ as a function of the action profile p . In the interesting cases (i.e., such that $p_k < 1, \forall k$), the Hessian of $U(p)$ is negative definite, thus $U(p)$ is concave. However, its first partial derivative with respect to p_k is not continuous in ϵ_k . If there exists user k 's action in $[0, p_{k,5}]$ such that $\frac{\partial U(p)}{\partial p_k} = 0$, then it is the maximizer for $U(p)$. Otherwise, either (1) $\frac{\partial U(p)}{\partial p_k} > 0$ for $p_k < \epsilon_k$ and $\frac{\partial U(p)}{\partial p_k} < 0$ for $p_k > \epsilon_k$, thus the maximizer is ϵ_k , or (2) $U(p)$ increases in p_k until reaching a maximum in $p_k = p_{k,5}$. ■

B. Intervention design

Once the intervention scheme is given, the interaction among users can be modeled with the game in Eq. (11), where

$$U_i^I(p) = \theta_i \ln \left[p_i \mathbb{E} \left[[r_i (p_i + n_i - \tilde{p}_i)]_0^1 \right] \prod_{j=1, j \neq i}^n (1 - p_j) \right]$$

and the expectation operator, $\mathbb{E}[\cdot]$, yields the average intervention level.

Lemma 4. Any \bar{p}_k such that $2\epsilon_k \leq \bar{p}_k \leq 1 - \epsilon_k$ can be made the unique non trivial NE of the game Γ^I by choosing $r_k \rightarrow +\infty$ and $\tilde{p}_k = \bar{p}_k + \epsilon_k$.

Proof: Due to space constraints, we only provide a sketch of the proof. We refer the interested reader to [13] for the complete proof. If $p_i < \bar{p}_i$, the intervention level is always equal to 0. If $p_i > \bar{p}_i + 2\epsilon_i$, the intervention level is always equal to 1. If $\bar{p}_i \leq p_i \leq \bar{p}_i + 2\epsilon_i$, the intervention might be 0 or 1, depending on the value of the estimation error n_i . The resulting average intervention level is $\frac{p_i - \bar{p}_i}{2\epsilon_i}$. With respect to p_i , $U_i(p)$ is increasing in $[0, \bar{p}_i]$ and $\frac{\partial U_i(p)}{\partial p_i}$ is decreasing in $[\bar{p}_i, \bar{p}_i + 2\epsilon_i]$. Thus, a necessary and sufficient condition such that \bar{p}_i is a global maximum is that $\frac{\partial U_i(p)}{\partial p_i} \leq 0$ for $p_i \rightarrow \bar{p}_i^+$, which results in $\bar{p}_i \geq 2\epsilon_i$. ■

Lemma 4 states that, using an extreme rule, each user k has the incentive to adopt a transmission probability \bar{p}_k which is ϵ_k lower than \tilde{p}_k , to avoid the possibility of an intervention triggered by the estimation errors. This is true as long as \tilde{p}_k is not too low, otherwise for user k it is convenient to adopt a transmission probability closer to \tilde{p}_k , accepting the risk of an intervention triggered by the estimation errors.

Proposition 4. If $p_k^* = \frac{\theta_k}{\sum_{i=1}^n \theta_i} \geq 2\epsilon_k$ for every user k , then the intervention rule $r_k \rightarrow +\infty$ and $\tilde{p}_k = p_k^* + \epsilon_k$ is an optimal affine intervention.

Proof: According to Lemma 4, users have the incentive to adopt $p^* = (p_1^*, \dots, p_n^*)$. In this case the intervention level is equal to 0 because the estimation errors can not be higher than $\epsilon = (\epsilon_1, \dots, \epsilon_n)$. Thus, the outcome of the system is equal to the benchmark optimum. Finally, this implies that $r_k \rightarrow +\infty$ and $\tilde{p}_k = p_k^* + \epsilon_k$ define an optimal affine intervention rule, and, in fact, also an optimal intervention rule within the class of all intervention rules. ■

Corollary 2. If $p_k^* = \frac{\theta_k}{\sum_{i=1}^n \theta_i} \geq 2\epsilon_k$, the optimal affine intervention rule is optimal in the class of all intervention rules.

We consider the following affine intervention rule, for every user k

$$r_k \rightarrow +\infty, \quad \tilde{p}_k = \begin{cases} p_k^* + \epsilon_k & \text{if } p_k^* \geq 2\epsilon_k \\ 3\epsilon_k & \text{otherwise} \end{cases} \quad (18)$$

If $p_k^* \geq 2\epsilon_k$ for every user k , (18) defines an optimal intervention rule. If $p_k^* < 2\epsilon_k$, for some user k , the intervention rule might not be optimal. This rule is designed with the objective to minimize the intervention level. In fact, each user i has the incentive to adopt the action $\tilde{p}_i - \epsilon_i$, which results in an intervention level equal to 0.

C. Comparison between pricing and intervention

Here we investigate the impact of imperfect monitoring on the performance attainable with the pricing and the intervention schemes. We consider the symmetric case, i.e., $\theta_i = \theta_j$ and $\epsilon_i = \epsilon_j, \forall i, j \in \mathcal{N}$. Thus, the optimal transmission policy in the cooperative scenario, defined by Eq. (4), is $p_k^* = \frac{1}{n}$, for every user k .

First we analyze how the social welfare and the total throughput vary increasing the number of users in the system. Fig. 2 shows that the estimation errors have different effects in the two schemes. The social welfare of the pricing scheme is not affected a lot by imperfect monitoring, in particular the total throughput and the social welfare when the number of users is less than or equal to $\frac{1}{\epsilon_i} = 10$ (corresponding to the condition $p_k^* \geq \epsilon_i$) are equal to those attainable in the perfect monitoring case. In fact, in this situation each user is (on average) charged correctly. The impact of the estimation errors in the intervention scheme is much stronger. The intervention scheme is able to achieve the optimal social welfare as long as the number of users is less than or equal to 5 (corresponding to the condition $p_k^* \geq 2\epsilon_k$, as predicted by Proposition 4). If the number of users is higher than 5, both the total throughput and the social welfare decrease rapidly as the number of users increases. This trend is a consequence of the action adopted by the users in this situation, which is constant and equal to $2\epsilon_k$ instead of scaling with the number of users. This causes a rapid increase of the number of collisions. This trend determines a threshold in the number of users such that, for a number of users lower than the threshold, intervention outperforms pricing, whereas, for a number of users higher than the threshold, pricing outperforms intervention. The threshold value for the considered system parameters is equal to 15.

In Fig. 3 the value of the threshold is plotted varying ϵ_k , the maximum intensity of the noise. The threshold decreases as ϵ_k increases, because the intervention scheme is more sensitive to the estimation errors than the pricing scheme. For the highest noise considered, i.e., $\epsilon_i = 0.2$, the intervention scheme outperforms the pricing scheme as long as the number of users is less than 9.

Finally, in Figs. 4 and 5 we compare the considered intervention scheme of Eq. (18) with the optimal affine intervention rule, which is computed through an exhaustive search (this is computationally possible because we consider a symmetric scenario). Fig. 4 shows the action selected by the users and the average intervention level varying the number of users,

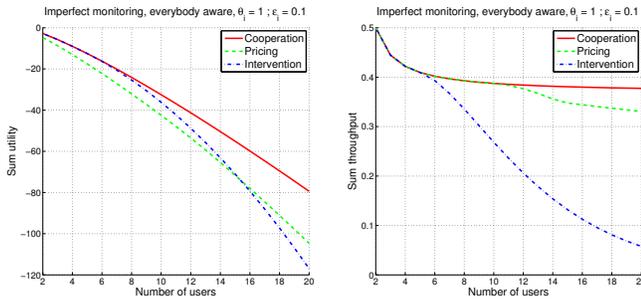


Fig. 2. Social welfare and total throughput vs. number of users, in the imperfect monitoring scenario

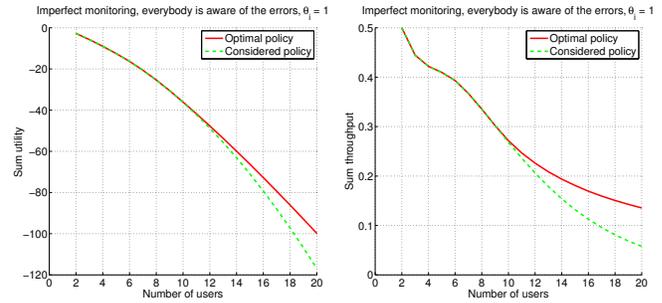


Fig. 5. Social welfare and total throughput vs. number of users in the imperfect monitoring scenario, considered vs. optimal policy

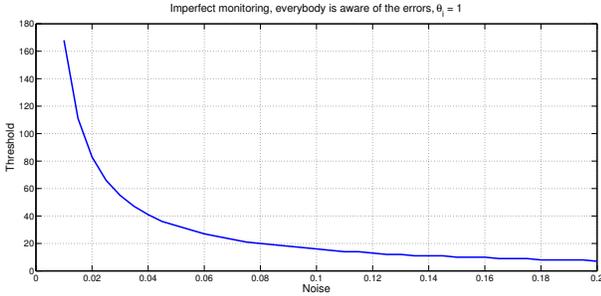


Fig. 3. Threshold vs. noise in the imperfect monitoring scenario

while Fig. 5 shows the social welfare and the total throughput varying the number of users. Proposition 4 guarantees that the considered intervention rule is optimal for a number of users equal to or lower than 5 (corresponding to the condition $p_k^* \geq 2\epsilon_k$). However, as we can see, the considered intervention rule is optimal until 9 users. If the number of users exceeds 9, it is preferable to be more aggressive with the intervention rule, using a \tilde{p}_k lower than $3\epsilon_k$ and forcing the users to decrease their transmission probability as well, even though this means that intervention is occasionally triggered.

VI. CONCLUSIONS

In this paper we tackle the problem of designing pricing and intervention schemes to provide incentives for the users to exploit efficiently the channel resource in a contention game. We have considered both the perfect monitoring and the imperfect monitoring scenarios. The analysis shows that

the intervention scheme, differently from the pricing scheme, is able to achieve the optimal performance in the perfect monitoring scenario. On the other hand, in the imperfect monitoring scenario, as a rough general principle, intervention achieves greater efficiency than pricing when the number of users is small, while the opposite is true when the number of users is large. The analysis in this paper can serve as a guideline for a designer to select between pricing and intervention and to design the best policy for the selected scheme, depending on some system parameters such as the number of users and the intensity of the monitoring noise.

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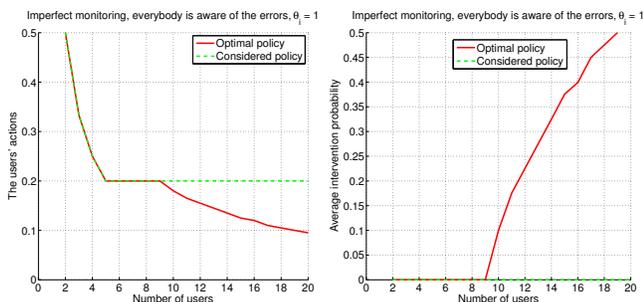


Fig. 4. The users' actions and the average level of intervention vs. number of users in the imperfect monitoring scenario, considered vs. optimal policy