Bidirectional Energy Trading and Residential Load Scheduling with Electric Vehicles in the Smart Grid

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***Abstract*—Electric vehicles (EVs) will play an important role in the future smart grid because of their capabilities of storing electrical energy in their batteries during off-peak hours and supplying the stored energy to the power grid during peak hours. In this paper, we consider a power system with an aggregator and multiple customers with EVs and propose novel electricity load scheduling algorithms which, unlike previous works, jointly consider the load scheduling for appliances and the energy trading using EVs. Specifically, we allow customers to determine how much energy to purchase from or to sell to the aggregator while taking into consideration the load demands of their residential appliances and the associated electricity bill. We propose two different approaches: a collaborative and a non-collaborative approach. In the collaborative approach, we develop an optimal distributed load scheduling algorithm that maximizes the social welfare of the power system. In the non-collaborative approach, we model the energy scheduling problem as a non-cooperative game among self-interested customers, where each customer determines its own load scheduling and energy trading to maximize its own profit. In order to resolve the unfairness between heavy and light customers in the non-collaborative approach, we propose a tiered billing scheme that can control the electricity rates for customers according to their different energy consumption levels. In both approaches, we also consider the uncertainty in the load demands, with which customers' actual energy consumption may vary from the scheduled energy consumption. To study the impact of the uncertainty, we use the worst-case-uncertainty approach and develop distributed load scheduling algorithms that provide the guaranteed minimum performances in uncertain environments. Subsequently, we show when energy trading leads to an increase in the social welfare and we determine what are the customers' incentives to participate in the energy trading in various usage scenarios including practical environments with uncertain load demands.**

***Index Terms*—Bidirectional energy trading; residential load scheduling; electric vehicles; load demand uncertainty; optimization theory; game theory**

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I. Introduction

Due to many benefits such as environmentally-friendly energy source, lower maintenance cost, and energy efficiency, interests in electric vehicles (EVs) have increased for the past few years. Moreover, compared to conventional hybrid electric vehicles (HEVs), evolved EVs, such as battery electric vehicles (BEVs) and plug-in hybrid electric vehicles (PHEVs), have enlarged battery capacities and intelligent converters so that they can not only charge but also discharge their batteries [3]. As EVs are being widely used and deployed, they are expected to lead to two significant impacts on the grid. First, due to the considerable amount of their energy consumption, EVs generate a significant amount of new load on the grid while being charged. This can cause serious problems such as a large capacity requirement for the increased peak demand, which causes underutilization of power plants during off-peak hours [4]. A natural solution to mitigate this problem is to optimize EVs' charging schedule with coordination, as shown in [5], [6], and [7]. Second, EVs can contribute to the grid by storing electrical energy in their batteries during off-peak hours and supplying the stored energy to the power grid during peak hours as distributed energy storages for the power grid. With the support of an aggregator, a group of customers with EVs can establish an energy trading market in which each customer can buy and sell energy through the aggregator to improve the system-wide performance (e.g., social welfare) as well as their own benefits.

Existing studies on energy trading among EVs have been mainly focused on coordinated charing/discharging only among EVs, e.g., [8], [9], [10], and [11]. In [8], the authors considered an energy trading market in which a number of EVs buy or sell energy from/to the aggregator. The energy trading problem was modeled as a non-cooperative game among EVs and a linear price function was proposed. In [9], the authors also considered an energy trading market using a non-cooperative game among EVs. Assuming multiple sellers and buyers, they used double auction to model the market clearing price function. Unlike [8] and [9] that considered single time-slot models, energy trading scheduling problems were studied spanning multiple time-slots in [10] and [11]. In [10], a coordinated charging/discharging scheduling algorithm was developed for a number of EVs aiming at minimizing the total cost at the aggregator. In [11], price uncertainty was considered for the energy trading market, where the energy trading price was assumed to change dynamically every hour based on a Markov chain.

Besides the coordinated charing/discharging only among EVs, the load scheduling of residential appliances can be jointly considered to further realize the potential benefits of EVs. As one of the key ingredients of the smart grid, residential load scheduling and demand response have been attracted significant research interests, e.g., [12], [13], [14], [15], [16], and [17]. If EVs are appropriately incorporated into load scheduling, they can be utilized to further improve the overall satisfaction on the load demands and/or reduce the electricity cost by consuming their stored energy for residential appliances. Prior studies on the incorporation of EVs into the residential load scheduling problems include [18], [19], and [20]. In [18] and [19], load scheduling problems were considered aiming at maximizing the social welfare of the power system, in which each customer owns an EV with charging/discharging capabilities. In [20], a load scheduling problem was studied in a single home model with an EV. In particular, in [17] and [20], the authors considered the load demand uncertainty due to the variation of load demands or the distortion introduced by the communication channel, which is one of the key challenges in load scheduling. Although, in the above works, EVs were considered as a type of appliance with the capability of charging and discharging, the use of discharged energy was limited within each customer's own household without allowing energy trading among the customers. If the customers are allowed to supply their energy stored in their EVs' batteries to the power system, the discharged energy can be more efficiently utilized to achieve a higher utility and/or a lower electricity cost.

In this paper, we study the energy scheduling problem for a power system that consists of an aggregator and multiple customers with EVs. Compared to the previous works in which residential load scheduling and EV-based energy trading were studied separately, we jointly consider load scheduling for residential appliances and bidirectional energy trading among customers by allowing the customers to buy and sell energy from/to the aggregator using their EVs. We propose two different approaches: a collaborative and a non-collaborative approach. In the collaborative approach, we formalize a social welfare maximization problem for the power system. By solving this problem, we develop a distributed energy scheduling algorithm that can achieve the optimal load scheduling and energy trading, thereby maximizing the social welfare of the power system. In the non-collaborative approach, we study the impact of the customers' non-cooperative behaviors on energy trading and the associated performance. To this end, we model the energy scheduling problem as a non-cooperative game among self-interested customers, where each customer determines its load scheduling and energy trading to maximize its own profit. Moreover, in order to resolve the unfairness between heavy and light customers, we propose a tiered billing scheme that can control the electricity rates for customers according to their different energy consumption levels. We show that the proposed game has a unique Nash equilibrium and develop a distributed algorithm that converges to the equilibrium. Finally, the uncertainty issue is investigated for both approaches. By assuming that the actual energy consumption may vary from the scheduled energy consumption, we investigate the impact of the uncertainty in load demands and develop distributed energy scheduling algorithms based on the worst-case-uncertainty approach. For various usage scenarios, we study the benefits from energy trading and incentives to participate in the energy trading as well as the impact of the load demand uncertainty. We summarize the comparison of our work with the existing works on electricity load scheduling in Table I.

TABLE I
COMPARISON WITH RELATED WORKS ON ELECTRICITY LOAD SCHEDULING.($\surd$: CONSIDERED, -:NOT CONSIDERED)

	Electric Vehicles	Residential Load Scheduling	Bidirectional Energy Trading	Load Demand Uncertainty
[8]-[11]	$\surd$	–	$\surd$	–
[12]-[16]	–	$\surd$	–	–
[17]	–	$\surd$	–	$\surd$
[18], [19]	$\surd$	$\surd$	–	–
[20]	$\surd$	$\surd$	–	$\surd$
Our work	$\surd$	$\surd$	$\surd$	$\surd$

The main contributions of this paper are as follows: (i) we focus on the bidirectional energy trading that is incorporated into residential energy scheduling problem in the power system, which has not been done before to the best of our knowledge; (ii) we develop distributed energy scheduling algorithms that can be implemented with the limited information through the communication network in the smart grid system; (iii) the impact of energy trading is studied in two different systems (collaborative and non-collaborative systems); (iv) a tiered billing scheme is proposed that resolves the unfairness between customers in the non-collaborative system; (v) considering the uncertainty in the load demands, we develop distributed energy scheduling algorithms that provide robust performance in the uncertain load demands.

The rest of the paper is organized as follows. In Section II, the system model is presented. In Sections III and IV, we define the energy scheduling problems and develop the distributed algorithms for collaborative and non-collaborative approaches, respectively. In Section V, the uncertainty in the load demands is investigated. In Section VI, we provide numerical results and finally we conclude in Section VII.

II. SYSTEM MODEL

We consider an electric power system which consists of a set of customers $\mathcal{I}$ and one aggregator as in Fig. 1. The aggregator buys electrical energy from a utility company through a wholesale electricity market and provides it to its customers. Each customer $i \in \mathcal{I}$ is equipped with the energy control system (ECS) and owns a number of residential appliances. Furthermore, each customer may have an EV. The ECS in each household can communicate with the aggregator as well as the appliances within the household. Through the ECS, each customer can control the scheduling of energy consumption for each of its appliances and EV. The entire scheduling interval (e.g., one day) is divided into T time-slots with equal duration, whose set is denoted by $\mathcal{T} = \{1, 2, \cdots, T\}$ (e.g., 24 time-slots each of which has one-hour duration). We assume that the scheduling of energy consumption is determined at the beginning of the entire scheduling interval (e.g., 0:00 AM). In

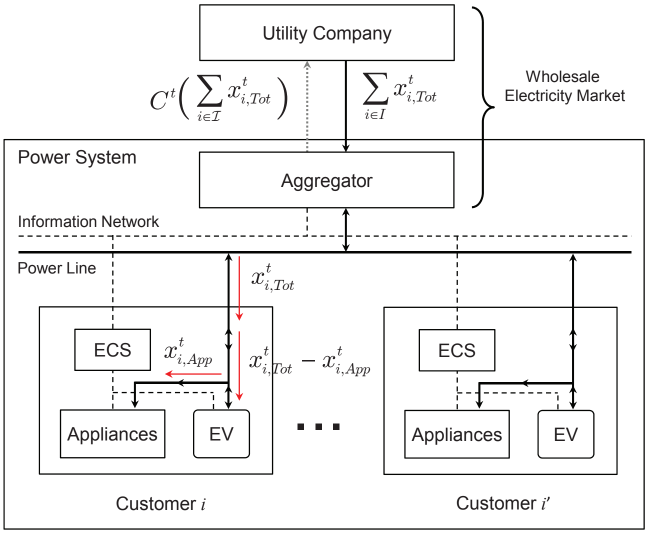


Fig. 1. Electric power system.

this paper, focusing on the energy trading among customers, we consider a simplified appliance model where the amount of energy consumed by all the residential appliances of customer i during time-slot t is denoted by a single variable $x^t_{i,App}$.[1] Then, we model customer i's utility function for its residential appliances during time-slot t as an increasing concave function $U^t_i(x^t_{i,App})$ of the amount of energy consumed by the residential appliances, $x^t_{i,App}$. Compared to residential appliances that only consume energy, the EV has the capability to charge and discharge energy into/from its built-in battery. The discharged energy can be used by other appliances, and thus $x^t_{i,App}$ includes the energy consumed from the battery in the EV as well as the aggregator. We denote the total amount of customer i's energy consumption during time-slot t by $x^t_{i,Tot}$ with the inclusion of the energy consumed by both the residential appliances and the EV. Accordingly, the difference between $x^t_{i,Tot}$ and $x^t_{i,App}$, i.e., $x^t_{i,Tot} - x^t_{i,App}$, is the amount of energy charged/discharged into/from the EV's battery during time-slot t. Positive difference, $x^t_{i,Tot} - x^t_{i,App} > 0$, represents that the corresponding amount of energy is charged to the EV's battery during time-slot t, while negative difference, $x^t_{i,Tot} - x^t_{i,App} < 0$, represents that the corresponding amount of energy is discharged from the EV's battery. We denote the overall energy scheduling of customer i by $\bar{x} = [\bar{x}_i]_{i\in\mathcal{I}}$, where $\bar{x}_i = [x^t_{i,Tot}, x^t_{i,App}]_{t\in\mathcal{T}}$.

We allow the bidirectional energy trading between customers and the aggregator, i.e., each customer can sell its energy to the aggregator when it has surplus energy stored in its EV's battery. Due to the energy trading capability, $x^t_{i,Tot}$ may have a negative value, which means that the corresponding amount of energy is sold to the aggregator. For example, if $x^t_{i,App} = 5$ kWh and $x^t_{i,Tot} = -10$ kWh (accordingly $x^t_{i,Tot} - x^t_{i,App} = -15$ kWh), customer i dishcarges 15 kWh of energy from its battery to use 5 kWh for its own appliances and to sell 10 kWh to the aggregator at t-th time-slot.

Due to physical constraints, we assume that there exist both maximum and minimum amounts of energy which the residential appliances and the customer i's household can consume during each time-slot, i.e.,

$$x^{min,t}_{i,App} \le x^t_{i,App} \le x^{max,t}_{i,App}, \quad \forall t \in \mathcal{T} \tag{1}$$

$$x^{min,t}_{i,Tot} \le x^t_{i,Tot} \le x^{max,t}_{i,Tot}, \quad \forall t \in \mathcal{T}. \tag{2}$$

The amount of charged (or discharged) energy of the EV during each time-slot is also constrained by the maximum and minimum values, i.e.,

$$x^{min,t}_{i,EV} \le x^t_{i,Tot} - x^t_{i,App} \le x^{max,t}_{i,EV}, \quad \forall t \in \mathcal{T}. \tag{3}$$

Since EVs may not be connected to the power grid throughout the day for various reasons (e.g., driving on the road), we assume that each customer's EV has a set of non-overlapping charging intervals, $\mathcal{L}_i = \{[S_{i,l}, F_{i,l}] | 1 \le l \le L_i, F_{i,l} \le S_{i,l+1}\}, \forall i \in \mathcal{I}$, where L_i denotes the number of charging intervals and $[S_{i,l}, F_{i,l}]$ denotes the l-th charging interval of customer i's EV. During the charging intervals, i.e., from the beginning of $S_{i,l}$-th time-slot to the end of $F_{i,l}$-th time-slot, the EV is connected to the power system, and thus can be either charged or discharged. The EV's battery charge level at the end of time-slot t is represented as

$$r^t_i = \begin{cases} v_{i,l}(1-\epsilon_i) + x^{S_{i,l}}_{i,Tot} - x^{S_{i,l}}_{i,App} & \text{if } t = S_{i,l} \\ r^{t-1}_i(1-\epsilon_i) + x^t_{i,Tot} - x^t_{i,App} & \text{if } S_{i,l} < t \le F_{i,l}, \end{cases}$$
$$\forall t \in [S_{i,l}, F_{i,l}], l \in \mathcal{L}_i, \tag{4}$$

where $v_{i,l}$ and $0 < \epsilon_i \ll 1$ denote the initial battery charge level at the beginning of charging interval l and the self-discharge rate of customer i's battery, respectively. For each time-slot, the EV's battery charge level is subject to the requirement on the minimum battery charge level and cannot exceed its battery capacity, as indicated below

$$r^{min,t}_i \le r^t_i \le R_i, \quad \forall t \in [S_{i,l}, F_{i,l}], l \in \mathcal{L}_i, \tag{5}$$

where $r^{min,t}_i$ and R_i denote the required minimum battery charge level at time-slot t and the battery capacity of customer i's EV, respectively.

For each time-slot, the aggregator buys electricity, which corresponds to the total demand of the power system, $\sum_{i\in\mathcal{I}} x^t_{i,Tot}$, from the utility company in the wholesale electricity market as illustrated in Fig. 1.[2] The utility company charges the aggregator an electricity cost based on a linear wholesale electricity price function per unit energy[3] [21], which is given by

$$p^t\Big(\sum_{i\in\mathcal{I}} x^t_{i,Tot}\Big) = a^t \sum_{i\in\mathcal{I}} x^t_{i,Tot} + b^t, \quad \forall t \in \mathcal{T}, \tag{6}$$

where $a^t > 0$ and $b^t > 0$ may vary in time and are predetermined by the utility company before the scheduling interval. We see from (6), as the total demand of the power

[1]Note that our system model and the following results in the rest of this paper can be easily extended to a more general model where each appliance's energy consumption is separately considered.

[2]In this paper, we assume that the customers in the power system have sufficiently large amount of load demands at each time-slot, and thus we do not consider the cases where the total demand of the power system has a negative value.

[3]For the convenience, 'wholesale electricity price per unit energy' will be represented by 'wholesale price' in the rest of this paper.

system increases, the wholesale price also increases. Then, we can represent the total cost of the aggregator at time-slot t as

$$C^t\Big(\sum_{i\in\mathcal{I}} x^t_{i,Tot}\Big) = p^t\Big(\sum_{i\in\mathcal{I}} x^t_{i,Tot}\Big) \times \sum_{i\in\mathcal{I}} x^t_{i,Tot} \quad \forall t \in \mathcal{T}. \tag{7}$$

III. Energy Scheduling: Collaborative Approach

In this section, we study the energy scheduling problem for a collaborative system, where all customers are willing to cooperate to maximize the social welfare of the system, which is defined as the sum of all customers' utilities minus the total cost charged to the aggregator. The resulting social welfare serves an upper bound on social welfare obtained using any feasible approaches.

We first consider the following social welfare maximization problem:

$$\begin{aligned} \mathbf{P}: \quad & \max_{\bar{x}} \quad \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} U^t_i(x^t_{i,App}) - \sum_{t\in\mathcal{T}} C^t\Big(\sum_{i\in\mathcal{I}} x^t_{i,Tot}\Big) \\ & \text{s.t.} \quad \bar{x}_i \in \mathcal{X}_i, \qquad \forall i \in \mathcal{I}, \end{aligned}$$

where $\mathcal{X}_i$ represents the feasible set of customer i's energy scheduling which is defined as a set of $\bar{x}_i$'s that satisfy conditions in (1),(2),(3), and (5). It can be easily shown that problem $\mathbf{P}$ is convex programming. Moreover, the objective function is strictly concave with respect to $\bar{x}_i$, the scheduling decision of any customer i, when the other customers' scheduling decisions, $\bar{x}_{-i} = [\bar{x}_{i'}]_{i'\neq i}$, are fixed. Due to these properties of problem $\mathbf{P}$, we can use the Gauss-Seidel method to solve it [22]. Denoting the objective function of problem $\mathbf{P}$ by $\phi(\bar{x}_1, \cdots, \bar{x}_{|\mathcal{I}|})$, the Gauss-Seidel method is shown below

$$\bar{x}^{(k+1)}_i = \underset{\bar{x}_i\in\mathcal{X}_i}{\operatorname{argmax}}\, \phi(\bar{x}^{(k+1)}_1, \bar{x}^{(k+1)}_{i-1}, \bar{x}_i, \bar{x}^{(k)}_{i+1}, \cdots, \bar{x}^{(k)}_{|\mathcal{I}|}), \quad \forall i \in \mathcal{I}, \tag{8}$$

where $|\mathcal{Z}|$ denotes the cardinality of set $\mathcal{Z}$. At each iteration $k+1$, customers update there energy scheduling decisions, $\bar{x}^{(k+1)}_i$, sequencially by solving the following problem:

$$\begin{aligned} \mathbf{P1}_i \quad & \max_{\bar{x}_i} \quad \sum_{t\in\mathcal{T}} U^t_i(x^t_{i,App}) - \sum_{t\in\mathcal{T}} C^t(x^t_{i,Tot} + X^{t,(k+1)}_{-i}) \\ & \text{s.t.} \quad \bar{x}_i \in \mathcal{X}_i, \end{aligned}$$

where $X^{t,(k+1)}_{-i} = \sum_{i'<i} x^{t,(k+1)}_{i',Tot} + \sum_{i'>i} x^{t,(k)}_{i',Tot}$. Problem $\mathrm{P1}_i$ is a convex optimization problem and can be solved by using standard algorithms for convex optimization. Hence, the proposed algorithm in (8) can be conducted in a distributed manner. Note that at each iteration, each customer needs to acquire only the information about the wholesale price function $p^t(\cdot), \forall t \in \mathcal{T}$ and the sum energy consumption of other customers, $X^{t,(k+1)}_{-i}, \forall t \in \mathcal{T}$. Such information can be updated and broadcast by the aggregator through the communication with the customer's ECS at the begging of each iteration.

We now study some properties of the optimal solution of problem $\mathbf{P}$ and introduce a new interpretation on it.

Proposition 1. *The optimal solution of problem $\mathbf{P}$, $\bar{x}^*$, can be represented as the collection of optimal solutions of the following problem:*

$$\begin{aligned} \mathbf{P2}_i \quad & \max_{\bar{x}_i} \quad \sum_{t\in\mathcal{T}} U^t_i(x^t_{i,App}) - \sum_{t\in\mathcal{T}} \pi^{*t} x^t_{i,Tot} \\ & s.t. \quad \bar{x}_i \in \mathcal{X}_i, \end{aligned}$$

where $\bar{\pi}^* = [\pi^{*t}]_{t\in\mathcal{T}}$ is given as

$$\pi^{*t} = C'^t\Big(\sum_{i\in\mathcal{I}} x^{*t}_{i,Tot}\Big) = 2a^t \sum_{i\in\mathcal{I}} x^{*t}_{i,Tot} + b^t, \quad \forall t \in \mathcal{T}, \tag{9}$$

and function $C'^t(z)$ denotes the first derivative of the total cost function $C^t(z)$ with respect to z.

Proof: See Appendix. ■

From Proposition 1, we can interpret problem $\mathbf{P2}_i, \forall i \in \mathcal{I}$ as a distributed energy scheduling framework based on a linear billing scheme where the aggregator decides the electricity rates $\bar{\pi}^* = [\pi^{*t}]_{\forall t\in\mathcal{T}}$, and apply these electricity rates to all customers. Then, each customer i decides its own energy scheduling to maximize its own utility by solving problem $\mathbf{P2}_i$. The aggregator charges each customer an electricity bill according to the announced electricity rates and the amount of energy consumption at each time-slot.

Proposition 2. *The sum of bills charged to all customers calculated based on the linear rate $\bar{\pi}^*$ in problem $\mathbf{P2}_i$, is always larger than or equal to the total cost which the aggregator has to pay to the utility company.*

Proof: Comparing the total cost function in (7) and the sum of bills based on the linear rate in (9), we can conclude that the following inequality is always true.

$$\begin{aligned} \sum_{t\in\mathcal{T}} \pi^{*t} \sum_{i\in\mathcal{I}} x^{*t}_{i,Tot} &= \sum_{t\in\mathcal{T}} C'^t\Big(\sum_{i\in\mathcal{I}} x^{*t}_{i,Tot}\Big) \sum_{i\in\mathcal{I}} x^{*t}_{i,Tot} \\ &= \sum_{t\in\mathcal{T}} C^t\Big(\sum_{i\in\mathcal{I}} x^{*t}_{i,Tot}\Big) + \sum_{t\in\mathcal{T}} a^t \Big(\sum_{i\in\mathcal{I}} x^{*t}_{i,Tot}\Big)^2 \\ &\geq \sum_{t\in\mathcal{T}} C^t\Big(\sum_{i\in\mathcal{I}} x^{*t}_{i,Tot}\Big). \end{aligned} \tag{10}$$

■

According to Proposition 2, the linear billing scheme always guarantees that the aggregator can make a profit regardless of how much energy the customers consume. However, it should be noted that the optimal electricity rates, $\bar{\pi}^*$, are determined only focusing on maximizing the social welfare without consideration on how we distribute the total cost to customers and the aggregator. Moreover, the linear electricity rates, $\bar{\pi}^*$, which are identically applied to all the customers, can result in unfairness among customers with different energy consumption levels. The unfairness issue can be explained as follows. Since the wholesale price, $p^t(\sum_{i\in\mathcal{I}} x^{*t}_{i,Tot})$, increases as the total demand at time-slot t, $\sum_{i\in\mathcal{I}} x^{*t}_{i,Tot}$, increases, the optimal electricity rate, π^{*t}, also increases as $\sum_{i\in\mathcal{I}} x^{*t}_{i,Tot}$ increases as shown in (9). Hence, the presence of heavy customers with a large energy consumption drives up the electricity rates, which is unfair to light customers. In the next section, we shall take into consideration the cost allocation and unfairness issues for a non-collaborative scenario.

IV. Energy Scheduling: Non-Collaborative Approach

In this section, we turn to the non-collaborative system, where customers are self-interested and try to maximize their own profits through load scheduling and energy trading. To model the self-interested customers' behaviors in the power system, we formulate the energy scheduling problem as a non-cooperative game. We then develop a distributed algorithm that converges to the unique Nash equilibrium of the non-cooperative game.

First, we propose a tiered proportional billing scheme, where each customer is charged with an electricity bill that consists of two parts: base rate and penalized rate. Mathematically, the billing function can be expressed as follows:

$$C_i^t(x_{i,Tot}^t, \bar{x}_{-i,Tot}^t) = \begin{cases} C^t\Big(\sum_{i'\in\mathcal{I}} x_{i',Tot}^t\Big)\dfrac{x_{i,Tot}^t}{\sum_{i'\in\mathcal{I}} x_{i',Tot}^t} & \text{if } x_{i,Tot}^t \le x_{Avg}^t \\ C^t\Big(\sum_{i'\in\mathcal{I}} x_{i',Tot}^t\Big)\dfrac{x_{Avg}^t}{\sum_{i'\in\mathcal{I}} x_{i',Tot}^t} \\ \quad +\alpha C^t\Big(\sum_{i'\in\mathcal{I}} x_{i',Tot}^t\Big)\dfrac{x_{i,Tot}^t - x_{Avg}^t}{\sum_{i'\in\mathcal{I}} x_{i',Tot}^t} & \text{if } x_{i,Tot}^t > x_{Avg}^t, \end{cases} \quad \forall t\in\mathcal{T}, i\in\mathcal{I} \tag{11}$$

where $\bar{x}_{-i,Tot}^t = [x_{i',Tot}^t]_{i'\neq i}$ denotes the energy consumptions of all customers except for customer i's energy consumption at time-slot t, $x_{Avg}^t = \sum_{i'\in\mathcal{I}} x_{i',Tot}^t/|\mathcal{I}|$ denotes the average energy consumption of all customers at time-slot t, and $\alpha \ge 1$ is a constant penalizing factor which is determined a priori by the aggregator. Each customer is subject to two-tier electricity rates according to its own energy consumption, $x_{i,Tot}^t$ as well as the average energy consumption, x_{Avg}^t, for each time-slot. Specifically, when a customer's energy consumption is less than or equal to the average consumption, i.e., $x_{i,Tot}^t \le x_{Avg}^t$, the base rate is applied, which is represented as $C^t(\sum_{i'\in\mathcal{I}} x_{i',Tot}^t)/\sum_{i'\in\mathcal{I}} x_{i',Tot}^t$, and the bill is charged proportionally to the amount of energy consumption. On the other hand, when a customer's energy consumption is larger than the average consumption, i.e., $x_{i,Tot}^t > x_{Avg}^t$, the base rate is applied to the energy consumption below the average value, x_{Avg}^t, while the penalized rate is applied to the energy consumption beyond the average value, $x_{i,Tot}^t - x_{Avg}^t$. The penalized rate is obtained by multiplying the base rate by the penalizing factor, α, i.e., $\alpha C^t(\sum_{i'\in\mathcal{I}} x_{i',Tot}^t)/\sum_{i'\in\mathcal{I}} x_{i',Tot}^t$.

Here, the penalizing factor α has two meanings. First, as α increases, heavy customers, who consume more energy than the average consumption, will pay more per unit energy consumption compared to the light customers, who consume less energy than the average consumption. Hence, by adjusting α, the aggregator can control the degree of fairness between heavy and light customers. Second, α affects the aggregator's profit. For example, if $\alpha = 1$, the sum of bills charged to all customers is equal to the total cost that the aggregator has to pay to the utility company, whereas if $\alpha > 1$, the aggregator can make a profit by charging heavy customers a higher rate.

Based on the tiered billing scheme in (11), the payoff function of customer i is defined as

$$u_i(\bar{x}_i, \bar{x}_{-i}) = \sum_{t\in\mathcal{T}} U_i^t(x_{i,App}^t) - \sum_{t\in\mathcal{T}} C_i^t(x_{i,Tot}^t, \bar{x}_{-i,Tot}^t), \quad \forall i\in\mathcal{I}, \tag{12}$$

where $\bar{x}_{-i} = [\bar{x}_{-i,Tot}^t]_{t\in\mathcal{T}}$. We now define the non-cooperative energy scheduling game $\mathbf{G} = \{\mathcal{I}, \{\mathcal{X}_i\}_{i\in\mathcal{I}}, \{u_i\}_{i\in\mathcal{I}}\}$, where $\mathcal{I}$, $\{\mathcal{X}_i\}_{i\in\mathcal{I}}$, and $\{u_i\}_{i\in\mathcal{I}}$ denote the sets of players, strategy sets, and payoff functions, respectively. In a non-cooperative game, the most accepted concept is Nash equilibrium [23]. A Nash equilibrium is a state of a non-cooperative game where no player can improve its utility by changing its strategy, if the other players maintain their current strategies. In our energy scheduling game $\mathbf{G}$, the Nash equilibrium is defined as an energy scheduling profile $\bar{x}^* = [\bar{x}_i^*]_{i\in\mathcal{I}}$ that satisfies the following:

$$u_i(\bar{x}_i^*, \bar{x}_{-i}^*) \ge u_i(\bar{x}_i, \bar{x}_{-i}^*), \quad \forall \bar{x}_i \in \mathcal{X}_i, i\in\mathcal{I}. \tag{13}$$

Next, we prove the existence and uniqueness of Nash equilibrium in the game $\mathbf{G}$, which are two critical properties of non-cooperative game [23].

Proposition 3. *The game* $\mathbf{G}$ *has a Nash equilibrium.*

Proof: We can easily show that $C_i^t(x_{i,Tot}^t, \bar{x}_{-i,Tot}^t)$ *is a strictly convex function of* $x_{i,Tot}^t$*. Accordingly, the payoff function* $u_i(\bar{x}_i, \bar{x}_{-i})$ *is a concave function of* $\bar{x}_i$*. Then, by the definition of the concave* N*-person game and Theorem 1 in [24], game* $\mathbf{G}$ *is a concave* N*-person game and has a Nash equilibrium.* ■

Proposition 4. *The Nash equilibrium of game* $\mathbf{G}$ *is unique, and the following algorithm converges to the unique equilibrium,* $\bar{x}^*$*.*

$$\bar{x}_i^{(k+1)} = [\bar{x}_i^{(k)} + \delta^{(k)} \nabla_{\bar{x}_i^{(k)}} u_i(\bar{x}_i^{(k)}, \bar{x}_{-i}^{(k)})]_{\mathcal{X}_i}, \quad \forall i\in\mathcal{I}, \tag{14}$$

where $\delta^{(k)}$ *is a step size at the* k*-th iteration,* $\nabla_{\bar{z}}$ *denotes the subgradient with respect to* $\bar{z}$*, and* $[\bar{z}]_{\mathcal{Z}}$ *denotes the projection of* $\bar{z}$ *onto constraint set* $\mathcal{Z}$*.*

Proof: Due to the page limitation, we provide the outline of the proof. As in equation (3.9) in [24], we can define the following vector:

$$g(\bar{x}) = [\nabla_{\bar{x}_1} u_1(\bar{x}_1, \bar{x}_{-1}); \cdots; \nabla_{\bar{x}_{|\mathcal{I}|}} u_{|\mathcal{I}|}(\bar{x}_{|\mathcal{I}|}, \bar{x}_{-|\mathcal{I}|})] \tag{15}$$

and denote its Jacobian by $G(\bar{x})$*. If matrix* $[G(\bar{x}) + G(\bar{x})^T]$ *is negative definite, game* $\mathbf{G}$ *has a unique Nash equilibrium and the iterative algorithm in (14) converges to the equilibrium by Theorems 2, 6, and 10 in [24]. Although it is a tedious process, we can easily show that the matrix* $[G(\bar{x}) + G(\bar{x})^T]$ *is negative definite, which completes the proof.* ■

By Propositions 3 and 4, we see that the unique Nash equilibrium can be achieved using the iterative algorithm in (14). In the proposed algorithm, each customer updates its own energy scheduling decision and energy trading based on the subgradient $\nabla_{\bar{x}_i^{(k)}} u_i(\bar{x}_i^{(k)}, \bar{x}_{-i}^{(k)})$ at each iteration k in a distributed manner. Each customer needs to acquire the wholesale price function, $p^t(\cdot), \forall t \in \mathcal{T}$, the total amount of energy consumption, $\sum_{i' \in \mathcal{I}} x^{t,(k)}_{i',Tot}, \forall t \in \mathcal{T}$, and the average energy consumption $x^{t,(k)}_{Avg}, \forall t \in \mathcal{T}$ at each iteration. The above information can be updated at each iteration through the communication between the aggregator and the customer's ECS.

Remark 1. *The base rate and the penalized rate of the tiered billing scheme can be rewritten as*

$$\rho^t_{base}(\bar{x}^t) = \frac{C^t\Big(\sum_{i' \in \mathcal{I}} x^t_{i',Tot}\Big)}{\sum_{i' \in \mathcal{I}} x^t_{i',Tot}} = a^t \sum_{i \in \mathcal{I}} x^t_{i,Tot} + b^t, \quad \forall t \in \mathcal{T}, \quad (16)$$

$$\rho^t_{\alpha}(\bar{x}^t) = \alpha \frac{C^t\Big(\sum_{i' \in \mathcal{I}} x^t_{i',Tot}\Big)}{\sum_{i' \in \mathcal{I}} x^t_{i',Tot}} = \alpha\Big(a^t \sum_{i \in \mathcal{I}} x^t_{i,Tot} + b^t\Big), \quad \forall t \in \mathcal{T}, \quad (17)$$

respectively, where $\bar{x}^t = [x^t_{i,Tot}]_{i \in \mathcal{I}}$. Note that, if $\alpha = 1$, these two rates are the same. Let us assume that $\bar{x}^*$ is the optimal solution of problem $\mathbf{P}$ in the previous section and $\alpha = 1$. Then, by comparing the linear electricity rate, π^{*t}, in (9) and the tiered electricity rate, $\rho^t_{base}(\bar{x}^{*t})$, in (16), it can be easily shown that, π^{*t} is larger than $\rho^t_{base}(\bar{x}^{*t})$. The difference between those two rates indicates how the customers will behave in the two different systems. Specifically, when the amount of customer i's energy consumption has a positive value, i.e., $x^{*t}_{i,Tot} > 0$, it will have an incentive to buy more energy in the non-collaborative system due to the lower electricity rate, $\rho^t_{base}(\bar{x}^{*t})$. On the other hand, when the amount of customer i's energy consumption has a negative value, i.e., $x^{*t}_{i,Tot} < 0$, it will have less incentive to sell its energy in the non-collaborative system since the lower electricity rate, $\rho^t_{base}(\bar{x}^{*t})$, will bring it a smaller profit from the energy trading. From these results, we can anticipate that the customers in the non-collaborative system will consume more energy and conduct less energy trading than the customers in the collaborative system. Even if $\alpha > 1$, similar behaviors are expected for the light customers in the non-collaborative system with the base rates, while the heavy customers will decrease the amount of their energy consumption according to α.

V. Uncertainty in Load Demands

In the previous sections, we have assumed that the amount of energy consumption is scheduled at the beginning of the entire scheduling interval. Nevertheless, the actual energy consumption may vary due to unexpected changes in the load demands of customers, e.g., uncertain weather condition and human behavior. Hence, it is important to study the impact of the uncertainty in load demands and develop energy scheduling algorithms that are robust and provide the guaranteed minimum performance in the uncertain environments. In this section, to tackle the uncertainty in load demands, we use the worst-case uncertainty approach in which the uncertain variables are assumed to be bounded in a given uncertainty set and the aim is to maximize the performance by considering the worst case in the uncertainty set. The result of this approach can be interpreted as the "best immunized against uncertainty" solution to a problem with uncertain parameters [25].

We define the uncertain load variation in customer i's energy consumption for its residential appliances at time-slot t as e^t_i. Then, we can represent customer i's *actual energy consumption* for its residential appliances not including the EV as $x^t_{i,App} + e^t_i$. For simplicity, we assume that when the load variation occur, each customer change the total energy consumption by the same amount of energy as the load variation so that the total energy consumption of customer i at time-slot t is represented as $x^t_{i,Tot} + e^t_i$. We assume that the uncertain load variation is bounded in the following uncertainty set:

$$\mathcal{E} = \{\bar{e} | e^{min,t}_i \le e^t_i \le e^{max,t}_i, \ \forall i \in \mathcal{I}, t \in \mathcal{T}\}, \quad (18)$$

where $e^{min,t}_i$ and $e^{max,t}_i$ denote the minimum and maximum values of load variation e^t_i, respectively. The maximum and minimum values of each customer's load variation are approved by or at least known to the aggregator (e.g., through the history database).

A. Load Demand Uncertainty in the Collaborative System

In the collaborative system, we aim to maximize the worst-case performance which is defined as the minimum social welfare of the power system under all possible load variation $\bar{e} \in \mathcal{E}$ using *robust optimization* techniques [25]. We first define the *actual social welfare* of the power system which comes from the actual energy consumptions as

$$\phi(\bar{x}, \bar{e}) = \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} U^t_i(x^t_{i,App} + e^t_i) - \sum_{t \in \mathcal{T}} C^t\Big(\sum_{i \in \mathcal{I}} (x^t_{i,Tot} + e^t_i)\Big). \quad (19)$$

Then, we formulate a robust optimization problem as

$$\mathbf{RP}: \max_{\bar{x}} \min_{\bar{e} \in \mathcal{E}} \phi(\bar{x}, \bar{e}) \quad \text{s.t.} \quad \bar{x}_i \in \mathcal{X}_i, \quad \forall i \in \mathcal{I}.$$

We first solve the inner part of problem $\mathbf{RP}$ which aims to minimize the actual social welfare with respect to $\bar{e}$. We denote the worst-case load variation which minimizes the actual social welfare given energy scheduling $\bar{x}$ by $\bar{e}(\bar{x}) = [e^t_i(\bar{x})]^{t \in \mathcal{T}}_{i \in \mathcal{I}}$, i.e.,

$$\bar{e}(\bar{x}) = \operatorname*{argmin}_{\bar{e} \in \mathcal{E}} \phi(\bar{x}, \bar{e}). \quad (20)$$

Since the actual social welfare in (19), $\phi(\bar{x}, \bar{e})$, is a concave function of $\bar{e}$ given $\bar{x}$, there is at least one solution of the minimization that is on the boundary of $\mathcal{E}$. Hence, to find the worst-case load variation $\bar{e}(\bar{x})$ given $\bar{x}$, we can compare the social welfare of all cases in which the load variation of each customer, e^t_i, has either its minimum value $e^{min,t}_i$ or its maximum value $e^{max,t}_i$. Since load variation variables are not coupled with each other at each time-slot, we can find the worst-case load variation at each time-slot as

$$\bar{e}_t(\bar{x}) = \underset{\bar{e}_t\in\tilde{\mathcal{E}}_t}{\operatorname{argmax}}\Big[\sum_{i\in\mathcal{I}} U_i^t(x_{i,App}^t + e_i^t) - C^t\Big(\sum_{i\in\mathcal{I}}(x_{i,Tot}^t + e_i^t)\Big)\Big], \quad \forall t\in\mathcal{T}, \quad (21)$$

where $\bar{e}_t(\bar{x}) = [e_i^t(\bar{x})]_{\forall i\in\mathcal{I}}$ and $\tilde{\mathcal{E}}_t = \{\bar{e}_t | e_i^t = e_i^{min,t}$ or $e_i^t = e_i^{max,t}, \ \forall i\in\mathcal{I}\}$. The complexity for finding the worst-case load variation $\bar{e}^t(\bar{x})$ is $2^{|\mathcal{I}|}$ for each time-slot t, which exponentially increases with the number of customers. However, in the next section, we will propose a distributed energy scheduling algorithm for the non-collaborative system that has a polynomial complexity.

With the worst-case load variation $\bar{e}(\bar{x})$, which is obtained by solving (21), we can rewrite problem $\mathbf{RP}$ as

$$\mathbf{RP}': \ \max_{\bar{x}} \tilde{\phi}(\bar{x}) \quad \text{s.t. } \bar{x}_i\in\mathcal{X}_i, \qquad \forall i\in\mathcal{I},$$

where $\tilde{\phi}(\bar{x}) = \phi(\bar{x}, \bar{e}(\bar{x}))$. We call the optimal solution to problem $\mathbf{RP}$ the *robust optimal solution*.

Proposition 5. *Problem $\mathbf{RP}'$ is a convex optimization problem.*

Proof: The actual social welfare $\phi(\bar{x},\bar{e})$ is a concave function of $\bar{x}$ given $\bar{e}$. Then, according to equation (3.7) in [26], the minimum of $\phi(\bar{x},\bar{e})$ along with $\bar{e}\in\mathcal{E}$, i.e., $\phi(\bar{x},\bar{e}(\bar{x}))$, is concave with respect to $\bar{x}$. As the constraint sets $\mathcal{X}_i, \forall i\in\mathcal{I}$ remain convex, problem $\mathbf{RP}'$ is a convex optimization problem. ■

Due to the fact that customer i's utility function $U_i^t(x_{i,App}^t + e_i^t(\bar{x}))$ is not a function of only its own energy scheduling $\bar{x}_i$ but a function of $\bar{x}$ which is determined by other customers' scheduling decisions, we cannot develop a distributed energy scheduling algorithm based on the Gauss-Seidel method as in Section III, even though problem $\mathbf{RP}'$ is convex as shown in Proposition 5. However, we can use the subgradient projection algorithm to obtain the robust optimal solution as

$$\bar{x}^{(k+1)} = [\bar{x}^{(k)} + \delta^{(k)}\nabla_{\bar{x}^{(k)}}\tilde{\phi}(\bar{x})]_{\mathcal{X}}, \quad (22)$$

where $\nabla_{\bar{x}^{(k)}}\tilde{\phi}(\bar{x}) = [\nabla_{\bar{x}_1^{(k)}}\tilde{\phi}(\bar{x}), \cdots, \nabla_{\bar{x}_{|\mathcal{I}|}^{(k)}}\tilde{\phi}(\bar{x})]$ represents the subgradient of $\tilde{\phi}(\bar{x})$ at $\bar{x}^{(k)}$. Since each element of the subgradient can be represented by using each customer's utility function, i.e.,

$$\nabla_{\bar{x}_i^{(k)}}\tilde{\phi}(\bar{x}) = \nabla_{\bar{x}_i^{(k)}}\Big[\sum_{t\in\mathcal{T}} U_i^t(x_{i,App}^t + e_i^t(\bar{x})) - \sum_{t\in\mathcal{T}} C^t\Big(\sum_{i\in\mathcal{I}}(x_{i,Tot}^t + e_i^t(\bar{x}))\Big)\Big], \quad (23)$$

the subgradient projection algorithm in (22) can be represented as a combination of partial update processes at each customer as

$$\bar{x}_i^{(k+1)} = [\bar{x}_i^{(k)} + \delta^{(k)}\nabla_{\bar{x}_i^{(k)}}\tilde{\phi}(\bar{x})]_{\mathcal{X}_i}, \quad \forall i\in\mathcal{I}. \quad (24)$$

In the iterative algorithm in (24), customer i requires the information about its worst-case load variation, $e_i^t(\bar{x}^{(k)}), \forall t\in\mathcal{T}$, and the sum of worst-case load variation for other customers, $\sum_{i\in\mathcal{I}} e_i^t(\bar{x}^{(k)}), \forall t\in\mathcal{T}$ as well as the sum energy consumption of other customers at each iteration. The worst-case load variation $\bar{e}(\bar{x})$ is calculated by the aggregator at each iteration. Since each customer requires only the sum of worst-case load variation and the sum of energy consumption for other customers, the iterative algorithm in (24) can be conducted in a distributed manner at each customer i by updating those information through the communication network between the aggregator and customers.

B. Load Demand Uncertainty in the Non-Collaborative System

To study the impact of the load demand uncertainty on the selfish customers' behavior in the non-collaborative system, we assume that each customer's objective is to maximize its worst-case payoff, i.e., the minimum value of its payoff under all load variation $\bar{e}\in\mathcal{E}$. Such a formulation is refereed to as the *robust game* [27]. We first define the *actual payoff* of customer i as

$$u_i(\bar{x}_i, \bar{x}_{-i}, \bar{e}) = \sum_{t\in\mathcal{T}} U_i^t(x_{i,App}^t + e_i^t) - \sum_{t\in\mathcal{T}} C_i^t(x_{i,Tot}^t + e_i^t, \bar{x}_{-i,Tot}^t + \bar{e}_{-i}^t), \quad \forall i\in\mathcal{I}. \quad (25)$$

Each customer tries to maximize its worst-case payoff defined as

$$\tilde{u}_i(\bar{x}_i, \bar{x}_{-i}) = u_i(\bar{x}_i, \bar{x}_{-i}, \bar{e}(\bar{x}; i)) = \min_{\bar{e}\in\mathcal{E}} u_i(\bar{x}_i, \bar{x}_{-i}, \bar{e}), \quad (26)$$

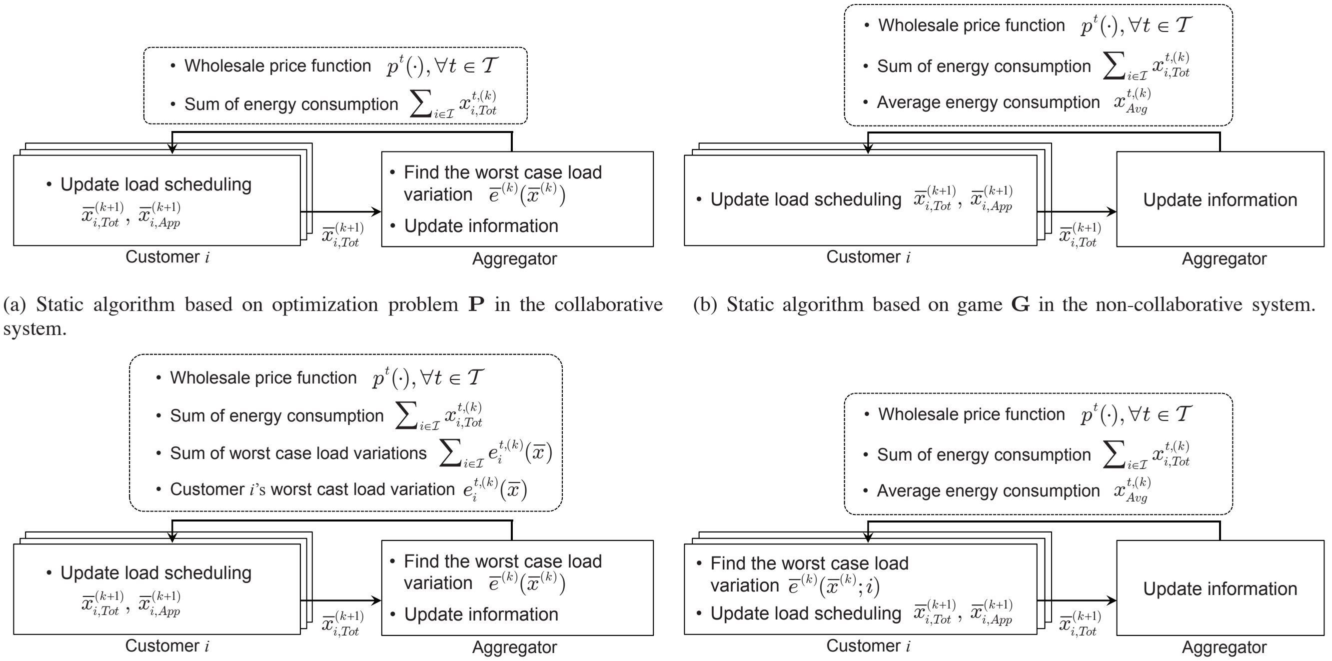
where $\bar{e}(\bar{x}; i)$ represents the worst-case load variation that minimizes customer i's payoff given $\bar{x}$, i.e.,

$$\bar{e}(\bar{x}; i) = \underset{\bar{e}\in\mathcal{E}}{\operatorname{argmin}}\, u_i(\bar{x}_i, \bar{x}_{-i}, \bar{e}). \quad (27)$$

Since the actual payoff of customer i in (25), $u_i(\bar{x}_i, \bar{x}_{-i}, \bar{e})$, is a concave function of $\bar{e}$ given $\bar{x}$, there is at least one solution to the minimization in (27) lying on the boundary of $\mathcal{E}$. Hence, to find the worst-case load variation for customer i, $\bar{e}(\bar{x}; i)$, given $\bar{x}$, we can compare customer i's payoffs by checking all the possible cases in which load variation of each customer, e_i^t, has either its minimum value $e_i^{min,t}$ or its maximum value $e_i^{max,t}$. Since load variation variables are not coupled with each other at each time-slot, we can find the worst-case load variation for customer i at each time-slot as

$$\bar{e}_t(\bar{x}; i) = \underset{\bar{e}_t\in\tilde{\mathcal{E}}_{t;i}}{\operatorname{argmax}} \sum_{t\in\mathcal{T}} U_i^t(x_{i,App}^t + e_i^t) - \sum_{t\in\mathcal{T}} C_i^t(x_{i,Tot}^t + e_i^t, \bar{x}_{-i,Tot}^t + \bar{e}_{-i}^t), \quad \forall t\in\mathcal{T}, \quad (28)$$

where $\bar{e}_t(\bar{x}; i) = [e_i^t(\bar{x}; i)]_{\forall i\in\mathcal{I}}$ and $\tilde{\mathcal{E}}_{t;i} = \{\bar{e}_t | e_i^t = e_i^{min,t}$ or $e_i^t = e_i^{max,t}, \forall i\in\mathcal{I}\}$. Note that we can represent customer i's actual payoff $u_i(\bar{x}_i, \bar{x}_{-i}, \bar{e})$ by using its own load variation e_i^t and the sum of other customers' load variations, $E_{-i}^t = \sum_{i'\neq i} e_{i'}^t$, at each time-slot t, i.e., $u_i(\bar{x}_i, \bar{x}_{-i}, \bar{e}_i, \bar{E}_{-i})$, where $\bar{e}_i = [e_i^t]_{t\in\mathcal{T}}$ and $\bar{E}_{-i} = [E_{-i}^t]_{t\in\mathcal{T}}$. This implies that to find the worst-case load variation for customer i, $\bar{e}_t(\bar{x}; i)$, it is sufficient to compare its payoffs of only four cases in



(a) Static algorithm based on optimization problem $\mathbf{P}$ in the collaborative system.

(b) Static algorithm based on game $\mathbf{G}$ in the non-collaborative system.

(c) Robust algorithm based on robust optimization problem $\mathbf{RP}$ in the collaborative system.

(d) Robust algorithm based on robust game $\mathbf{RG}$ in the non-collaborative system.

Fig. 2. Distributed implementation of the proposed energy scheduling algorithms.

which customer i's load variation, e_i^t, has either its minimum value $e_i^{min,t}$ or its maximum value $e_i^{max,t}$ and the sum of other customers' load variations, E_{-i}^t, has either its minimum value $\sum_{i'\neq i} e_{i'}^{min,t}$ (i.e., $e_{i'}^t = e_{i'}^{min,t}, \forall i' \neq i$) or its maximum value $\sum_{i'\neq i} e_{i'}^{max,t}$ (i.e., $e_{i'}^t = e_{i'}^{max,t}, \forall i' \neq i$). Hence, if each customer has the information about the maximum and minimum values of the sum of load variation in the power system, each customer can find the worst-case load variation for its own payoff in a distributed manner. The complexity for finding the worst-case load variation at each customer is only 4 for each time-slot, which is considerably lower than that of the algorithm for the collaborative system.

With the worst-case load variation $\bar{e}(\bar{x})$, we define robust game $\mathbf{RG} = \{\mathcal{I}, \{\mathcal{X}_i\}_{i\in\mathcal{I}}, \{\tilde{u}_i\}_{i\in\mathcal{I}}\}$, where $\mathcal{I}$, $\{\mathcal{X}_i\}_{i\in\mathcal{I}}$, and $\{\tilde{u}_i\}_{i\in\mathcal{I}}$ denote the sets of players, strategy sets, and payoff functions, respectively. Moreover, we refer to the Nash equilibrium of robust game $\mathbf{RG}$ as *robust Nash equilibrium*.

Proposition 6. *Robust game* $\mathbf{RG}$ *has a unique robust Nash equilibrium and the following algorithm converges to the unique equilibrium,* $\bar{x}^*$.

$$\bar{x}_i^{(k+1)} = [\bar{x}_i^{(k)} + \delta^{(k)} \nabla_{\bar{x}_i^{(k)}} \tilde{u}_i(\bar{x}_i, \bar{x}_{-i})]_{\mathcal{X}_i}, \ \forall i \in \mathcal{I}. \quad (29)$$

Proof: Similarly to the proof of Proposition 5, we can easily show that payoff function $\tilde{u}_i(\bar{x}_i, \bar{x}_{-i})$ *is a concave function of* $\bar{x}_i$. *Then, we can apply the proofs of Propositions 3 and 4 to prove that robust game* $\mathbf{RG}$ *is a concave N-person game with a unique Nash equilibrium and the iterative algorithm in (29) converges to the equilibrium.* ■

According to the payoff function in (26), the subgradient of customer i's payoff $\nabla_{\bar{x}_i^{(k)}} \tilde{u}_i(\bar{x}_i, \bar{x}_{-i})$ can be calculated based on its local energy consumption information, $\bar{x}_i^{(k)}$, the sum of other customers' energy consumptions, $\sum_{i'\neq i} x_{i',Tot}^{t,(k)}, \forall t \in \mathcal{T}$, and the information of the worst-case load variation $\bar{e}(\bar{x}^{(k)}; i)$. As we mentioned, if customer i acquires the information about the maximum and minimum values of $\sum_{i'\neq i} e_{i'}^{t,(k)}(\bar{x}^{(t)}; i)$, it can calculate its own worst-case load variation $\bar{e}(\bar{x}^{(k)}; i)$, which enables the distributed implementation of the subgradient algorithm in (29). Those information can be updated by the aggregator at the beginning of each iteration.

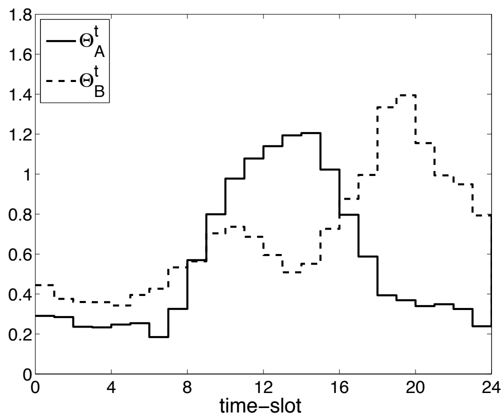
VI. Numerical Results

In this section, we provide numerical results to validate the proposed energy scheduling algorithms. Before addressing the simulation environments, we summarize the distributed implementation of our proposed algorithms in Fig. 2 including the operation of the aggregator and the customers at each iteration and required information exchange. We refer to the algorithms proposed in Sections III and IV without considering the load demand uncertainty as the *static algorithms*, while referring to the algorithms proposed in Section V which consider the load demand uncertainty as the *robust algorithms*.

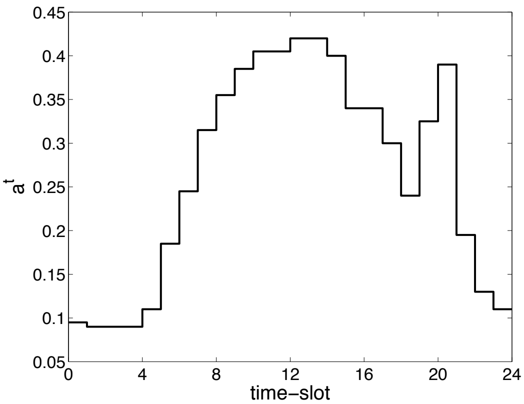
We consider the energy scheduling of one day and divide it into $T = 24$ time-slots, each of which corresponds to one hour. For illustration purpose, each customer's utility function is defined as

$$U_i^t(x_{i,App}^t) = \theta_i^t \log(1 + x_{i,App}^t), \quad (30)$$

where $\bar{\theta}_i = [\theta_i^t]_{t\in\mathcal{T}}$ represents customer i's hourly energy demand profile. In this paper, each customer has one of two different types of residential energy demand profiles, $\bar{\Theta}_A = [\Theta_A^t]_{t\in\mathcal{T}}$ and $\bar{\Theta}_B = [\Theta_B^t]_{t\in\mathcal{T}}$, as shown in Fig. 3(a), which are based on the average load shapes of commercial services and residential services, respectively [28].



(a) Two types of residential load demands profiles.



(b) Time varying coefficient a^t in the wholesale price function.

Fig. 3. Demand profiles and wholesale price function for the power system.

Each customer's minimum required energy consumption for its residential appliances at each time-slot t is set to $x^{min,t}_{i,App} = \theta^t_i/2$ kWh, $\forall t \in \mathcal{T}$. We assume that the coefficient a^t in the wholesale electricity price function varies hourly as in Fig. 3(b), whereas b^t is fixed to 0.05.

We consider three types of EVs:

- Type I : EV with only a chargeable battery;
- Type II : EV with a chargeable and dischargeable battery but no energy trading capability;
- Type III : EV with a chargeable and dischargeable battery and bidirectional energy trading capability.

If a customer has a Type I or II EV, it cannot sell energy to the aggregator, i.e., $x^{min,t}_{i,Tot} = 0$. On the other hand, if a customer has a Type III EV, it has the capability to sell the stored energy in its EV to the aggregator during its charging intervals, $\mathcal{L}_i$. Hence, in this case, the amount of total energy consumption at time-slot t can have a negative value, i.e., $x^{min,t}_{i,Tot} < 0$. If a customer has an EV with the dischargeable battery (i.e., Type II or III EV), the minimum amount of charged energy during a time-slot can have a negative value, i.e., $x^{min,t}_{i,EV} < 0$. Note that we can regard a the case with Type I EV as the reference case with a residential appliance which only consumes energy without discharging capability.

A. Impact of Energy Trading

In order to evaluate the benefits of bidirectional energy trading in the optimal energy scheduling, we first consider a simple scenario with two customers in a collaborative system. Customer 1 and customer 2 have the demand profiles $\bar{\Theta}_A$ and $\bar{\Theta}_B$, respectively. We assume that only customer 1 has an EV. Three different types of EVs, i.e., Type I, II, and III, are considered. For all cases, the EV's battery has a capacity of 30 kWh, i.e., $R_1 = 30$ kWh, and the maximum amount of charged energy is set to 5 kWh, i.e., $x^{max,t}_{1,EV} = 5$ kWh, $\forall t \in \mathcal{T}$. For Type I and II cases, the maximum amount of discharging energy is set to 5 kWh, i.e., $x^{min,t}_{1,EV} = -5$ kWh, $\forall t \in \mathcal{T}$. Finally, the maximum amount of energy that the customer with the Type III EV can sell to the aggregator is limited by 5 kWh, i.e., $x^{min,t}_{1,Tot} = -5$ kWh, $\forall t \in \mathcal{T}$. In this scenario, we assume that the EV's initial battery charge level and minimum required battery charge level are 0. The EV is assumed to be connected to the power system during the entire scheduling interval, i.e., $\mathcal{L}_1 = \{[1, 24]\}$.

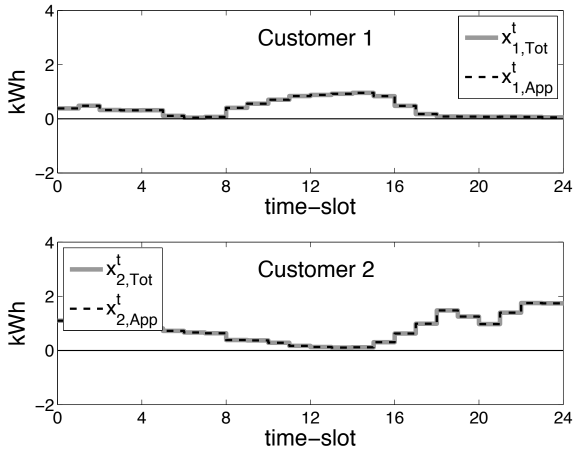
TABLE II
PERFORMANCE COMPARISON OF THREE TYPES OF EVS.

Type of EVs	Type I	Type II	Type III
Social welfare	13.69	17.98	18.65
Total utility	22.25	27.75	28.53
Customer 1's utility	8.08	13.49	12.30
Customer 2's utility	14.16	14.26	16.23
Total demand (kWh)	28.14	33.22	33.79
Total energy trading (kWh)	0	0	5.63
Total cost ($)	8.55	9.77	9.87
Average wholesale price ($/kWh)	0.30	0.29	0.29

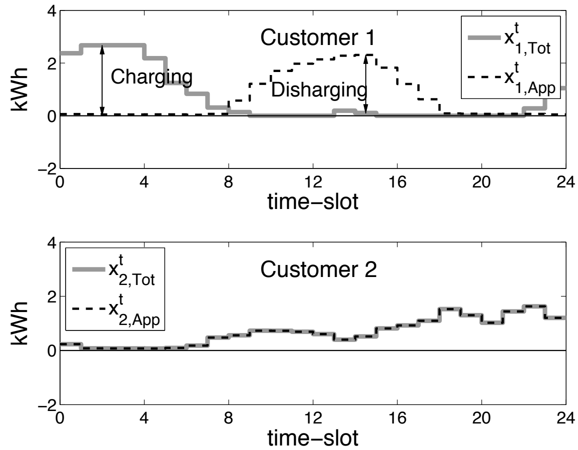
In Fig. 4, we compare the behaviors of two customers according to the type of customer 1's EV. Figs. 4(a), 4(b), and 4(c) show the energy consumptions of two customers when customer 1 has a Type I, II, and III EV, respectively. The solid lines represent the total energy consumed by each customer and the dashed lines represent the amount of energy consumed by each customer's residential appliances. The difference between those two values represents the amount of charged or discharged energy from/to the EV. The negative value of the total energy consumption in Fig. 4(c) represents the corresponding amount of energy is sold to the aggregator. Fig. 4(d) shows the wholesale price at which the aggregator buys energy from the utility company. Fig. 4(e) shows the total energy demand and the sum of energy consumed by the residential appliances in the power system. The shaded region represents the total amount of energy trading among customers and the aggregator. Performances of the power system and each customer are summarized in Table II. The average wholesale price is obtained by dividing the total cost by the total demand.

When customer 1 has a Type I EV, due to the lack of discharging capability, customers should buy energy for their appliances at each time-slot as in Fig. 4(a). Hence, during customers' peak hours (e.g., 8:00 to 16:00 for customer 1 and 16:00 to 24:00 for customer 2), the power system buys energy from the utility company at a relatively high wholesale price as shown in Fig. 4(d).

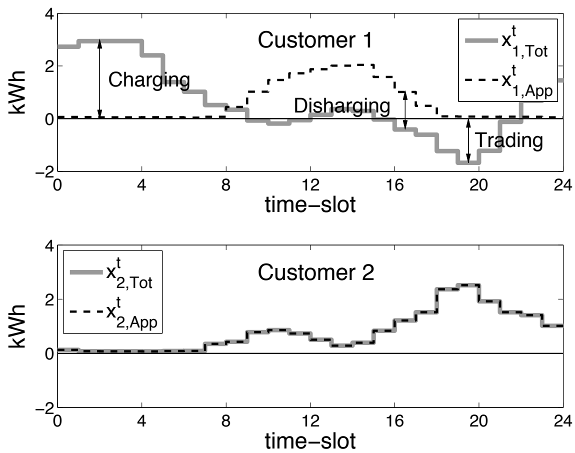
When customer 1 has a Type II EV, it can utilize its EV's battery as an energy storage by charging it during its off-peak hours, (e.g., 0:00 to 8:00), and discharging it during its peak hours, (e.g., 8:00 to 16:00), as in Fig. 4(b). Hence, it can use



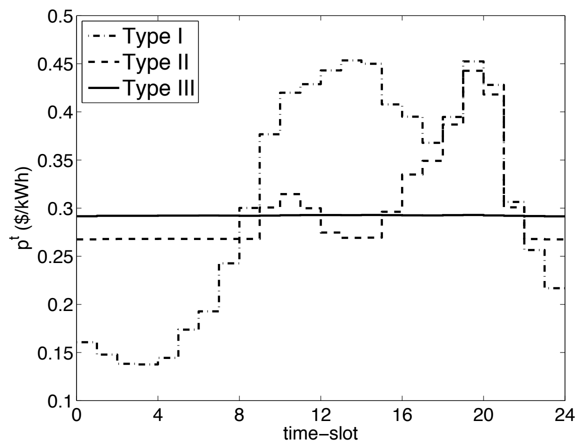
(a) Customers' behaviors with a Type I EV.



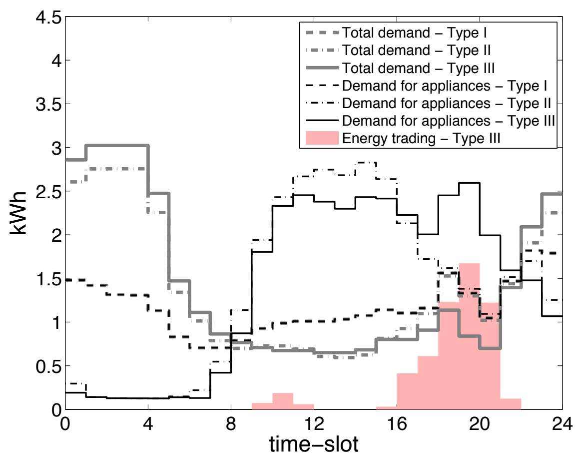
(b) Customers' behaviors with a Type II EV.



(c) Customers' behaviors with a Type III EV.



(d) Wholesale price of the power system.

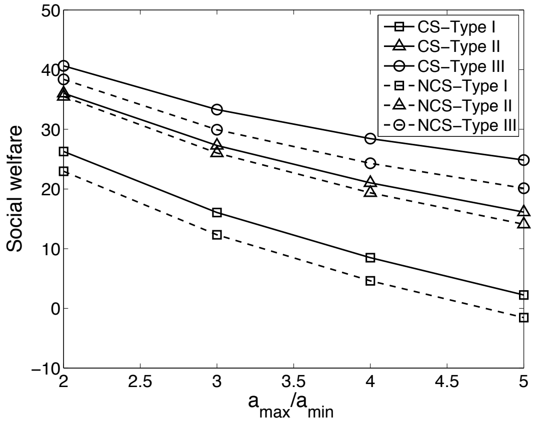


(e) Total energy consumption of the power system.

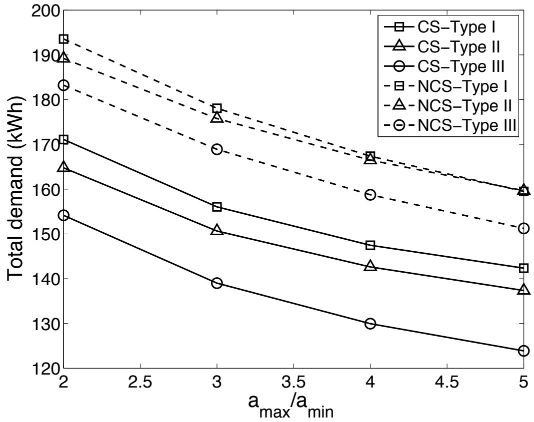
Fig. 4. Comparison of customers' behaviors according to different types of EVs in the collaborative system.

the stored energy for its appliances and does not need to buy energy during it's peak hours. As a result, as shown in Fig. 4(e), the power system buys more energy during the off-peak hours and buys less energy during customer 1's peak hours. As shown in Fig. 4(d), although the storage of energy raises the wholesale price of the off-peak hours, the power system can buy more energy at the lower wholesale price during customer 1's peak hours, which increases customer 1's utility as well as the social welfare as shown in Table II. However, it should be noted that the discharged energy from customer 1's EV can be used only for its own appliances due to the lack of the the energy trading mechanisms. Hence, the stored energy cannot be efficiently utilized during customer 2's peak hours, in which customer 2 has high load demands while customer 1 does not.

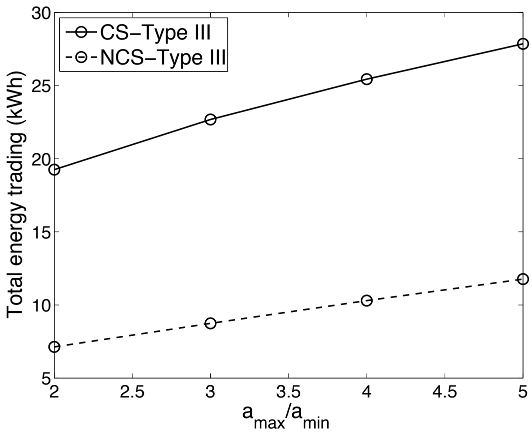
In the Type III case, customer 1 discharges energy from its EV and sells it to the aggregator during customer 2's peak hours (e.g., 16:00 to 24:00) as well as uses it for its own



(a) Average social welfare.



(b) Average total demand.



(c) Average total amount of energy trading.

Fig. 5. Performance comparison of collaborative system (CS) and non-collaborative system (NCS) varying a_{max}/a_{min}.

appliances during its peak hours as shown in Fig. 4(c). At the same time, customer 2 consumes more energy compared to the Type II case during its peak hours. Fig. 4(e) shows that with the energy trading, customer 1 uses its battery more flexibly than in the Type II case by charging more energy during the off-peak hours, which increases the wholesale price of the off-peak hours as shown in 4(d). On the other hand, during customer 2's peak hours, a large portion of the demand for appliances in the power system is supplied by customer 1's discharged energy, which reduces the total demand in

TABLE III
PERFORMANCE COMPARISON OF THE COLLABORATIVE SYSTEM (CS) AND THE NON-COLLABORATIVE SYSTEM (NCS).

Type of EVs	Type I CS / NCS	Type II CS / NCS	Type III CS / NCS
Social welfare	2.25 / -1.55	16.13 / 14.10	24.84 / 20.12
Total utility	53.15 / 63.12	61.37 / 75.30	61.49 / 75.11
Total demand (kWh)	142.36 / 159.50	137.36 / 159.67	123.87 / 151.23
Total energy trading (kWh)	0 / 0	0 / 0	27.86 / 11.77
Total cost ($)	50.90 / 64.68	45.24 / 61.21	36.64 / 55.00
Average wholesale price ($/kWh)	0.36 / 0.40	0.33 / 0.38	0.29 / 0.36
Sum of bills ($)	98.96 / 64.68	87.73 / 61.21	70.82 / 55.00

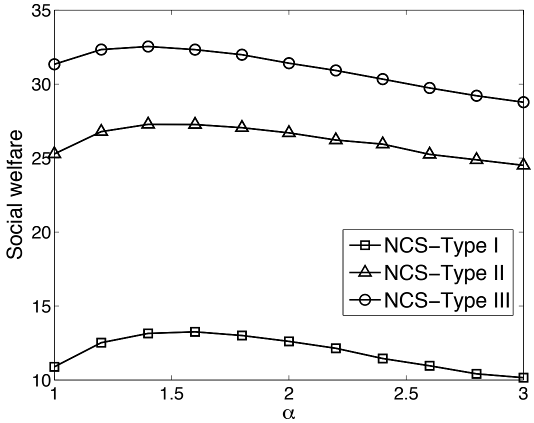
the power system. This enables the power system to buy more energy while keeping the wholesale price low during customer 2's peak hours as in shown Fig. 4(d). Although this increases the off-peak wholesale prices, the power system can use more energy for customer 2's appliances, which leads to further improvements in customer 2's utility, and thus the social welfare as shown in Table II. These results show that appropriate scheduling with energy trading can bring benefits to the power system, especially when the energy stored in a customer's EV can be efficiently utilized to satisfy other customers' load demands during their peak hours.

B. Comparison of the Collaborative and Non-Collaborative Approaches

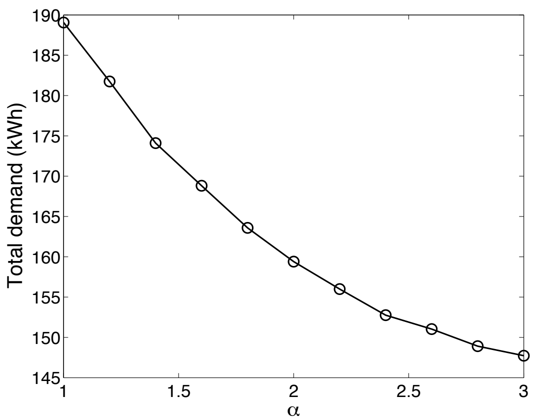
In this subsection, we compare the collaborative and non-collaborative systems. For each system, we consider three different cases, in each of which the customers have Type I, II, and III EVs, respectively. We consider 8 customers, each of which has a uniformly distributed energy demand profile taking one of two energy demand profiles $\bar{\Theta}_A$ and $\bar{\Theta}_B$ as its average value. We assume that the charging intervals of each customer are randomly determined with average connected duration of 10 hours and average disconnected duration of 5 hours. The battery capacity of customer i's EV, R_i is chosen randomly within $[15\text{kWh}, 40\text{kWh}]$. The minimum required charge level is chosen randomly within $[0.5R_i, 0.7R_i]$ only at the last time-slot of each charging interval. For the non-collaborative system, the penalizing factor α is set to 1. The results in this subsection are obtained by averaging 10 simulation results.

Table III shows that in both collaborative and non-collaborative systems, the power system benefits from energy trading (in terms of social welfare) compared to the cases without energy trading capabilities. Although the power system in the Type III case achieves almost the same total utility as in the Type II case, it consumes less amount of energy even at the lower wholesale price, and thus pays a less total cost. These results imply that energy trading enables the power system to more efficiently use the energy consumption for the customers' appliances as well as to buy the energy consumption at a lower wholesale price.

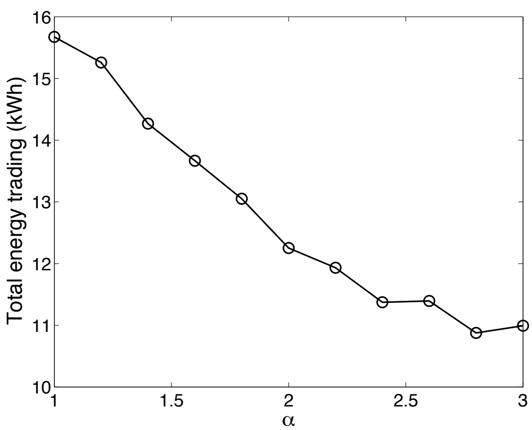
Table III also shows that, in the non-collaborative system, the customers have a tendency to consume more energy compared to that of the collaborative system. Moreover, in the Type III case, less amount of energy trading is conducted



(a) Average social welfare.



(b) Average total demand.



(c) Average total amount of energy trading.

Fig. 6. Performance of non-collaborative system (NCS) varying α.

among customers in the non-collaborative system than in the collaborative system. These results can be explained by the fact that for the same amount of energy consumption, the customers in the non-collaborative system have less incentives to participate in energy trading since the tiered proportional billing scheme leads to lower electricity rates than the optimal billing scheme in the collaborative system as discussed in Remark 1. Hence, energy scheduling in the non-collaborative system results in a higher utility and pay the more cost. Due to self-interested behaviors of customers, the power system in the non-collaborative system buys energy at a higher wholesale price, and thus achieves a lower social welfare than in the collaborative system. We can also observe that, in the collaborative system, the sum of electricity bills which are charged to customers based on the linear billing scheme is larger than the total cost of the aggregator, while those values are exactly the same in the non-collaborative system with the penalizing factor $\alpha = 1$ as discussed in Proposition 2.

C. Impact of the Wholesale Electricity Price

In order to study the impact of the wholesale price function $p^t(\cdot)$ on the energy scheduling, in Fig. 5, we compare the proposed energy scheduling algorithms with different values of $a^t, \forall t \in \mathcal{T}$ in the wholesale price function in (6). We adjust the ratio of the highest value to the lowest value of a^t, which is denoted by a_{max}/a_{min}, where $a_{max} = \max_{t\in\mathcal{T}} a^t$ and $a_{min} = \min_{t\in\mathcal{T}} a^t$, while maintaining the shape of time-varying α^t in Fig. 3(b) and letting a_{min} be fixed at 0.0225. Except for the wholesale price function, we use the same simulation settings as in subsection VI-B considering two different systems and three types of EVs. Fig. 5(a) shows the social welfare of six different cases by varying the ratio a_{max}/a_{min}. As expected, as the ratio a_{max}/a_{min} increases, the social welfare decreases in all cases. Fig. 5(b) shows that the total demand also decreases as a_{max}/a_{min} increases due to the increased wholesale price. However, with the Type III EVs, the power system always achieves the highest social welfare compared to the cases with the Type I and II EVs in both collaborative and non-collaborative systems. Moreover, we can observe from Fig. 5(c) that as the ratio a_{max}/a_{min} increases, both the amount of traded energy and the performance gap between the cases with Type II and III EVs increase. This implies that, as the wholesale price increases, energy trading plays a more important role in mitigating the negative impact of the wholesale price increase on the power system.

D. Impact of the Penalizing Factor α

In order to study the impact of the penalizing factor α of the tiered proportional billing scheme on energy scheduling, we compare in Fig. 6 the average performance of the non-collaborative system for different values of α. The ratio a_{max}/a_{min} is set to 3. Except for α and a_{max}/a_{min}, we use the same simulation settings as in subsection VI-B.

Fig. 6(a) shows the social welfare of three cases with different types of EVs varying α from 1 to 3. For all cases, as α increases, the social welfare increases up to a certain point and then decreases. This implies that we can improve the social welfare of the non-collaborative system by appropriately choosing α. However, as α increases, the customers monotonically reduce the amount of their energy consumption as well as the amount of energy trading, as shown in Figs. 6(b) and 6(c), since the increase in α raises the overall electricity rates of the power system.

In order to understand the impact of α on customers behaviors, we show the electricity rates of the tiered billing scheme and customers' energy consumptions. Since the three cases (Type I, II, and III) have similar tendencies according to α, we consider only the case with Type III EVs in Fig. 7. Fig. 7(a) shows the base electricity rates and the penalized rates versus time-slots for different values of α. Compared to the base rate when $\alpha = 1$, the penalized rates sharply increase as α increases, especially when a^t's in the wholesale price function are high. As we have shown in Fig. 6, due to the increased penalized rates, the total amount of energy consumption decreases, which in turn lowers the base rates. Accordingly, heavy customers are charged at higher rates than light customers as desired. In order to study the impact of α on individual customer's behavior, we show in Figs. 7(b) and 7(c) each individual customer's energy consumptions at a time-slot with low a^t (5:00) and at a time-slot with high a^t (14:00) by comparing the cases when $\alpha = 1$ and when $\alpha = 1.4$. In both time-slots, as α increases, the heavy customers tend to decrease their energy consumption due to the penalized rates, while the light customers tend to increase their energy consumption to take advantage of decreased base rates.

E. Impact of the Load Demand Uncertainty

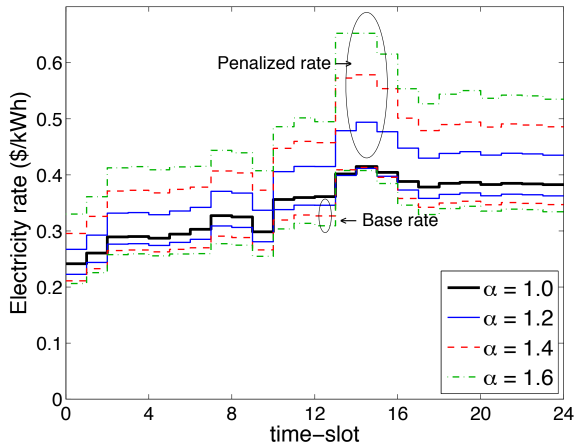
To study the impact of load demand uncertainty in both collaborative and non-collaborative systems, we compare in Figs. 8 and 9 the performance obtained by using the robust algorithms which consider load demand uncertainty to those obtained by the static algorithms which neglect load demand uncertainty. We set the maximum and minimum values of the uncertain energy consumption, $e_i^{max,t}$ and $e_i^{min,t}$, to be proportional to the minimum required energy consumption $x_i^{min,t}$, i.e.,

$$e_i^{max,t} = \omega \times x_i^{min,t}, \quad \forall t \in ,i \in \mathcal{I}, \qquad (31)$$

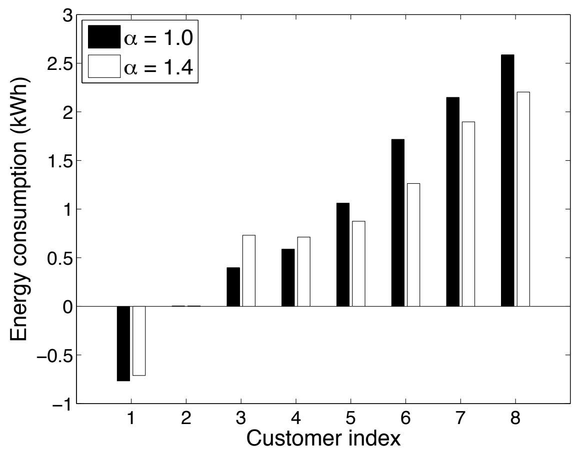
$$e_i^{min,t} = -\min(\omega, 1) \times x_i^{min,t}, \quad \forall t \in ,i \in \mathcal{I}, \qquad (32)$$

where ω represents the relative degree of the load demand uncertainty in the power system. The minimum operation in (32) represents that the energy consumption for residential appliances cannot be negative. The uncertain energy consumption e_i^t follows a uniform distribution between its maximum and minimum values. We use the same simulation settings as in subsection VI-B and consider only Type III EVs for brevity.

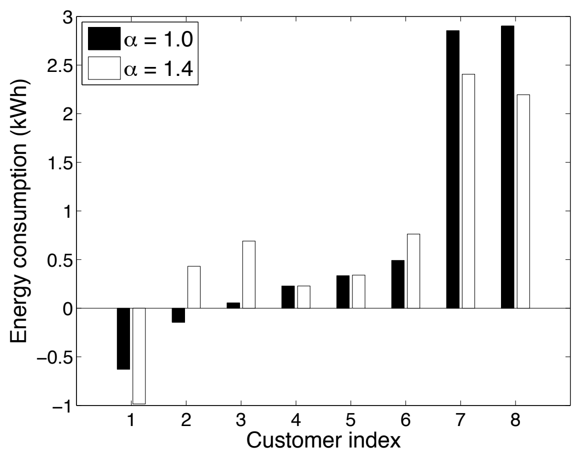
In Fig. 8, we compare the average performances of four algorithms by varying ω from 0 to 3. The result in Fig. 8 is obtained by repeating and averaging 1000 outcomes of the random load variation $\bar{e}$. Fig. 8(a) shows that, as ω increases, the average worst-case social welfare decreases in both collaborative and non-collaborative systems. However, the robust algorithms always achieve a higher worst-case social welfare than that of the static algorithms. This result means that we can improve the minimum guaranteed performance of the power system by using the robust optimization and game. Similarly, in Fig. 8(b), as ω increases, the standard deviation increases in both systems. The robust algorithms always have a lower standard deviation than that of the static algorithms, which means the energy scheduling obtained by the robust algorithms is more stable against the load demand uncertainty. In both worst-case social welfare and standard deviation, it is observed that the performance differences between the robust algorithms and the static algorithms increase as ω increases, which implies the importance of the robust algorithms particularly in a high uncertainty environment.



(a) Hourly price per unit energy consumption with different α.



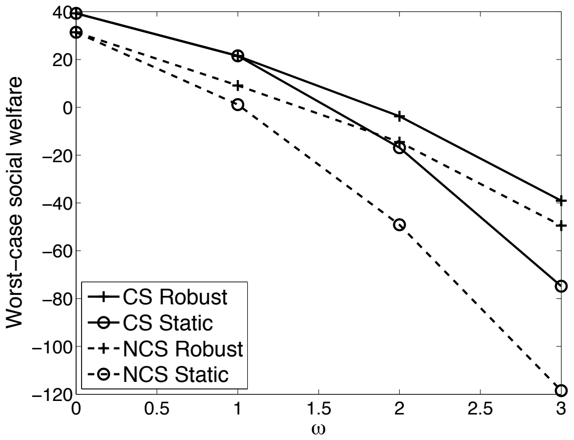
(b) Energy consumption of each customer during a time-slot with low a^t (5:00).



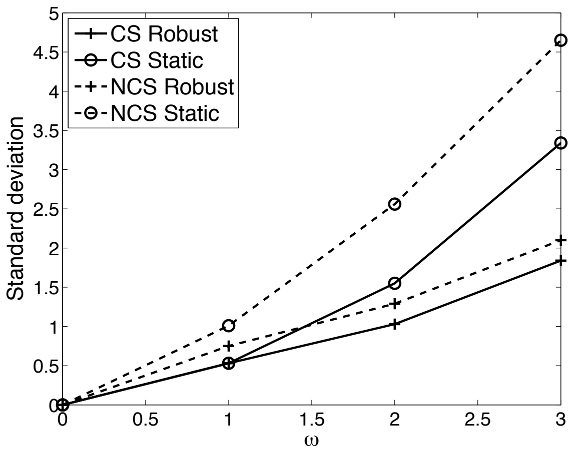
(c) Energy consumption of each customer during a time-slot with high a^t (14:00).

Fig. 7. Impact of α on hourly price and customers' energy consumption.

In order to show the effect of the robust algorithms in the practical environments, in Fig. 9, we show the actual social welfare according to each of outcomes of random load variation when $\omega = 3$. As in Figs. 9(a) and 9(b), in both collaborative and non-collaborative systems, we can see that the robust algorithms achieve the lower degree of variation in the social welfare than that of the static algorithms. This

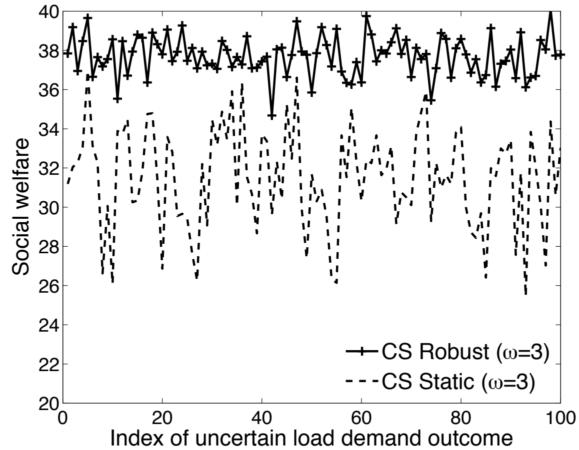


(a) Average worst-case social welfare.

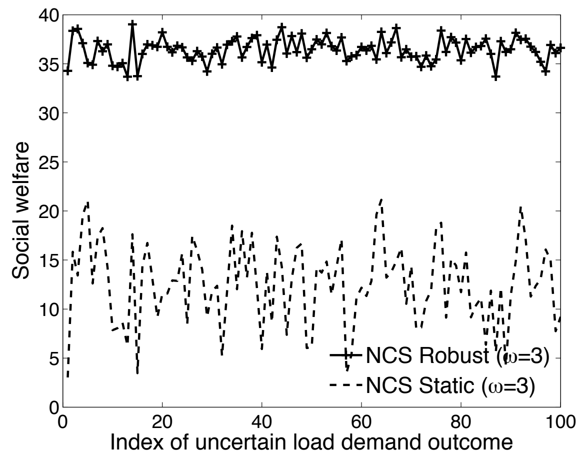


(b) Average standard deviation of social welfare.

Fig. 8. Performances comparison of the robust algorithms and the static algorithms varying ω.



(a) Collaborative system.



(b) Non collaborative system.

Fig. 9. Impact of the load demand uncertainty.

property of robust algorithms enables us to achieve stabilized energy scheduling in the presence of load demand uncertainty.

VII. Conclusion

In this paper, we studied an energy scheduling problem for the power system where bidirectional energy trading is allowed among the aggregator and customers by utilizing the charging and discharging capabilities of EVs. We proposed two different approaches: collaborative and non-collaborative approaches. In the collaborative approach, we developed a distributed energy scheduling algorithm that maximizes the social welfare of the power system. In the non-collaborative approach, we modeled a non-cooperative energy scheduling game among self-interested customers and proposed a tiered billing scheme to resolve the fairness issue between heavy and light customers. Then, we developed a distributed algorithm that converges to the unique Nash equilibrium of the energy scheduling game. For both the collaborative and non-collaborative approaches, a more practical system model was considered to capture the uncertain load demands. By using robust optimization and robust game theory, we developed two distributed scheduling algorithms yielding the optimal solution and Nash equilibrium that are robust against the load demand uncertainty.

Numerical results show that the appropriately scheduled energy trading can lead to a higher social welfare to the power system in both collaborative and non-collaborative systems. The social welfare improvement becomes more significant when the energy stored in customers' EVs can be utilized to satisfy other customers' load demands during their peak hours. Moreover, as the wholesale electricity price increases, energy trading plays a more important role in mitigating the negative impact of the increased wholesale electricity price. Comparing two systems and resulting the corresponding billing schemes, we show that customers in the non-collaborative system tend to consume more energy but sell less energy, since the tiered billing scheme in the non-collaborated leads to lower electricity rates than the optimal billing scheme in the collaborative system. The results also show that by appropriately adjusting the penalizing factor of the tiered billing scheme, we can effectively mitigate the unfairness among customers as well as improve the social welfare. Finally, we showed that in the presence of load demand uncertainty, the robust energy scheduling algorithm based on the worst-case uncertainty approach can achieve the improved performance in terms of the worst-case social welfare as well as reduce the degree of variation of the social welfare while providing stabilized energy scheduling to customers.

APPENDIX
PROOF OF PROPOSITION 1

We define the Lagrangian of problem $\mathbf{P}$ as

$$L(\bar{\lambda}, \bar{x}) = \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} U_i^t(x_{i,App}^t) - \sum_{t\in\mathcal{T}} C^t(\sum_{i\in\mathcal{I}} x_{i,Tot}^t) + \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}_i^t} \lambda_{i,j}^t h_{i,j}^t(\bar{x}_i), \quad (33)$$

where $\mathcal{J}_i^t$ denotes the set of indices for the inequality constraint functions of $\mathcal{X}_i$ associated to customer i at time-slot t, and $h_{i,j}^t(\bar{x}_i), \forall j \in \mathcal{J}_i^t, t \in \mathcal{T}$ and $\bar{\lambda} = [\lambda_{i,j}^t]_{j\in\mathcal{J}_i^t}^{t\in\mathcal{T}}$ denote the corresponding constraint functions and Lagrangian multipliers, respectively. Using the KKT condition, at the optimal solution of problem $\mathbf{P}$ in the previous section, $\bar{x}^*$, there exists a Lagrangian multipliers $\bar{\lambda}$ that satisfie the following condition set:

$$\mathcal{C}: \begin{cases} \dfrac{\partial L(\bar{\lambda}, \bar{x})}{\partial x_{i,App}^t} = {U'}_i^t(x_{i,App}^{*t}) + \displaystyle\sum_{j\in\mathcal{J}_i^t} \lambda_{i,j}^t \dfrac{h_{i,j}^t(\bar{x}_i)}{x_{i,App}^t} = 0, \\ \qquad\qquad\qquad\qquad\qquad\qquad \forall t \in \mathcal{T}, i \in \mathcal{I} \\ \dfrac{\partial L(\bar{\lambda}, \bar{x})}{\partial x_{i,Tot}^t} = -C'^t(\displaystyle\sum_{i\in\mathcal{I}} x_{i,Tot}^{*t}) + \sum_{j\in\mathcal{J}_i^t} \lambda_{i,j}^t \dfrac{h_{i,j}^t(\bar{x}_i)}{x_{i,Tot}^t} = 0, \\ \qquad\qquad\qquad\qquad\qquad\qquad \forall t \in \mathcal{T}, i \in \mathcal{I}. \end{cases}$$

Similarly, the Lagrangian of problem $\mathbf{P2}_i$ and the corresponding KKT condition set can be represented as

$$L_i(\bar{\lambda}, \bar{x}) = \sum_{t\in\mathcal{T}} U_i^t(x_{i,App}^t) - \sum_{t\in\mathcal{T}} \pi^{*t} x_{i,Tot}^t + \sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{J}_i^t} \lambda_{i,j}^t h_{i,j}^t(\bar{x}_i), \quad (34)$$

and

$$\mathcal{C}2_i: \begin{cases} \dfrac{\partial L_i(\bar{\lambda}, \bar{x})}{\partial x_{i,App}^t} = {U'}_i^t(x_{i,App}^{*t}) + \displaystyle\sum_{j\in\mathcal{J}_i^t} \lambda_{i,j}^t \dfrac{h_{i,j}^t(\bar{x}_i)}{x_{i,App}^t} = 0, \\ \qquad\qquad\qquad\qquad\qquad\qquad \forall t \in \mathcal{T} \\ \dfrac{\partial L_i(\bar{\lambda}, \bar{x})}{\partial x_{i,Tot}^t} = -\pi^t + \displaystyle\sum_{j\in\mathcal{J}_i^t} \lambda_{i,j}^t \dfrac{h_{i,j}^t(\bar{x}_i)}{x_{i,Tot}^t} = 0, \\ \qquad\qquad\qquad\qquad\qquad\qquad \forall t \in \mathcal{T}, \end{cases}$$

respectively. If we assume that we already know the optimal solution of problem $\mathbf{P}$, $\bar{x}^*$, and set $\pi^{*t} = C'^t(\sum_{i\in\mathcal{I}} x_{i,Tot}^{*t}), \forall t \in \mathcal{T}$, the collection of conditions $\mathcal{C}2_i, \forall i \in \mathcal{I}$ has the same form as condition $\mathcal{C}$. Hence, if $\bar{x}^*$ is the optimal solution of problem $\mathbf{P}$, it also always maximizes each problem $\mathbf{P2}_i$ under the condition that $\pi^{*t} = C'^t(\sum_{i\in\mathcal{I}} x_{i,Tot}^{*t}), \forall t \in \mathcal{T}$.

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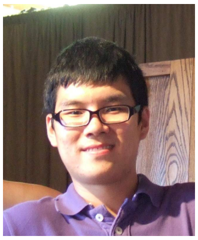

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