The Theory of Intervention Games for Resource Sharing in Wireless Communications

Jaeok Park and Mihaela van der Schaar, *Fellow, IEEE*

***Abstract*—This paper develops a game-theoretic framework for the design and analysis of a new class of incentive schemes called intervention schemes. We formulate intervention games, propose a solution concept of intervention equilibrium, and prove its existence in a finite intervention game. We apply our framework to resource sharing scenarios in wireless communications, whose non-cooperative outcomes without intervention yield suboptimal performance. We derive analytical results and analyze illustrative examples in the cases of imperfect and perfect monitoring. In the case of imperfect monitoring, intervention schemes can improve the suboptimal performance of non-cooperative equilibrium when the intervention device has a sufficiently accurate monitoring technology, although it may not be possible to achieve the best feasible performance. In the case of perfect monitoring, the best feasible performance can be obtained with an intervention scheme when the intervention device has a sufficiently strong intervention capability.**

***Index Terms*—Game theory, incentives, intervention, resource sharing, wireless communications.**

I. INTRODUCTION

WHEN self-interested users share resources non-cooperatively, it is common that the resources are utilized suboptimally from a global point of view [2]. Hence, overcoming the suboptimal performance of non-cooperative outcomes poses an important challenge for successful resource utilization. The aforementioned phenomenon is widely observed in wireless communications, where users compete for radio resources interfering with each other. For the sake of discussion, consider the following abstract scenario of resource sharing in communications. First, users determine their resource usage levels, which in turn determine the service quality they receive. In general, as the overall usage level increases, the service quality is reduced due to interference or congestion. The payoff of a user is determined by its own usage level as well as the service quality. In such a scenario, users tend to choose a higher usage level than the socially optimal one. That is, it is in the self-interest of users to choose a high usage level, although reducing their usage levels simultaneously would benefit all of them. In game theory, such a conflict between private and social interests is modeled as the prisoner's dilemma game. In the literature, it has been shown that various wireless communication scenarios exhibit a prisoner's dilemma phenomenon, including packet forwarding [3], distributed spectrum allocation [4], and medium access control (CSMA/CA [5] and slotted Aloha [6]).

Incentive schemes are needed to improve the performance of non-cooperative outcomes. In this paper, we propose a class of incentive schemes based on the idea of intervention. Implementing an intervention scheme requires an intervention device that is able to monitor the actions of users and to affect their resource usage. An intervention manager first chooses an intervention rule used by the intervention device, and then users choose their actions knowing the intervention rule chosen by the manager. After observing a signal about the actions of users, the intervention device chooses its action according to the intervention rule. The manager chooses an intervention rule to maximize his payoff, anticipating the rational behavior of users given the intervention rule. The payoff of the manager can be considered as a measure of the system performance, which can incorporate various efficiency and fairness criteria. We formulate the interaction between users and a manager as an intervention game and propose a solution concept called intervention equilibrium. Intervention equilibrium predicts the outcome of an intervention game in terms of an intervention rule chosen by the manager and an operating point chosen by users.

Intervention can be classified into two types, called type 1 and type 2, depending on how the intervention device acts in the system relative to users. In type-1 intervention, the intervention device acts in a symmetric way as users do while having the ability to monitor the actions of other users. An example of type-1 intervention can be found in [7] and [8], which consider a random access network where an intervention device interferes with other users by transmitting its packets after obtaining information about the transmission probabilities of users. In type-2 intervention, the intervention device acts as a gatekeeper which can control the service quality received by users. An example of type-2 intervention can be found in [9] and [10]. [9] analyzes scheduling mechanisms where a scheduler assigns different priorities to traffic flows depending on their input rates, and [10] considers a packet dropping mechanism where the server determines the probability of dropping packets as a function of the total arrival rate. The two types of intervention can be applied to the aforementioned resource sharing scenario, as schematically shown in Fig. 1.

The goal of intervention schemes to improve the performance of non-cooperative outcomes is illustrated in Fig. 2

Manuscript received 15 January 2011; revised 15 July 2011. A preliminary version of this paper was presented in part at the 30th IEEE International Conference on Computer Communications (IEEE INFOCOM 2011), Shanghai, China, Apr. 2011 [1].

J. Park is with the School of Economics, Yonsei University, Seoul 120-749, Korea (e-mail: jaeok.park@yonsei.ac.kr). This work was done when J. Park was with the Electrical Engineering Department, University of California, Los Angeles (UCLA), Los Angeles, CA 90095, USA.

M. van der Schaar is with the Electrical Engineering Department, University of California, Los Angeles (UCLA), Los Angeles, CA 90095, USA (e-mail: mihaela@ee.ucla.edu).

Digital Object Identifier 10.1109/JSAC.2012.120115.

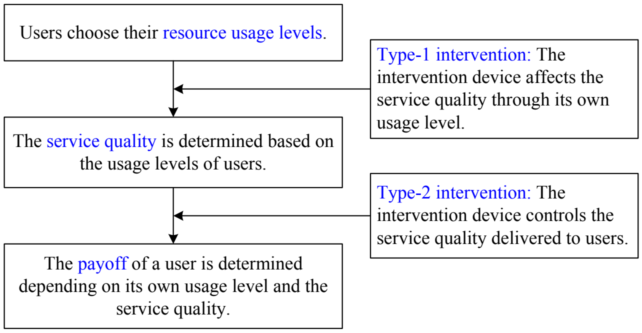


Fig. 1. Two types of intervention in a resource sharing scenario.

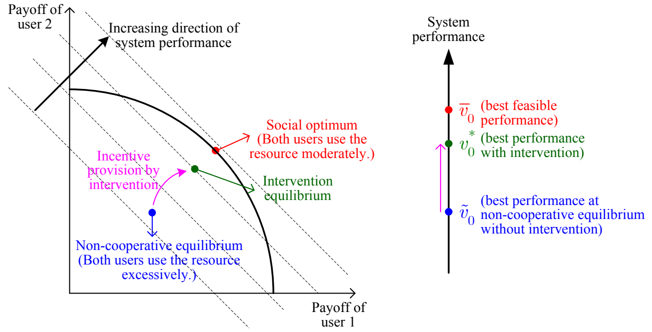


Fig. 2. Performance improvement through an intervention scheme. (The system performance is given by the average payoff, and a dotted line represents the set of payoff profiles that yield the same system performance.)

with two users and the system performance measured by the average payoff of the two users. Our analysis is aimed at answering the following two questions.

1) When can we construct an intervention scheme that improves the suboptimal performance of non-cooperative equilibrium?
2) When can we construct an intervention scheme that achieves the best feasible performance?

Our analysis suggests that the answers to these questions depend on the ability of the intervention device:

- Ability to monitor the actions of users (i.e., monitoring technology),
- Ability to affect the payoffs of users through its actions (i.e., intervention capability).

The discussion on the example in Section III-B shows that an intervention scheme can improve the performance of non-cooperative equilibrium when the monitoring technology is sufficiently accurate. This result is reinforced by the analytical results and the example in Section IV, which considers the case of perfect monitoring. The analytical result in Section III-A shows that intervention schemes may not achieve the best feasible performance when the monitoring technology is noisy. On the other hand, the analytical results and the example in Section IV show that intervention schemes can achieve the best feasible performance when monitoring is perfect and the intervention device has a sufficiently strong intervention capability. When signals are noisy, the manager can provide incentives by triggering a punishment following signals that are more likely to occur when users deviate. When these signals occur with positive probability even when users do not deviate, punishment happens from time to time at equilibrium, which results in a performance loss. On the contrary, when signals are perfectly accurate, punishment through intervention can be used only as a threat, which is never used at equilibrium. Thus, in the case of perfect monitoring, it is possible for the manager to achieve a desired operating point without incurring a performance loss.

The rest of this paper is organized as follows. In Section II, we formulate intervention games, develop a solution concept of intervention equilibrium, and show its existence in a finite intervention game. In Sections III and IV, we derive analytical results and discuss illustrative examples in the cases of imperfect and perfect monitoring, respectively. In Section V, we compare intervention schemes with existing approaches in the literature. In Section VI, we conclude.

II. Intervention Games and Intervention Equilibrium

We consider a system (e.g., a wireless network) where N users and an intervention device interact. The set of the users is finite and denoted by $\mathcal{N} = \{1, \dots, N\}$. The action space of user i is denoted by A_i, and a pure action for user i is denoted by $a_i \in A_i$, for all $i \in \mathcal{N}$. A pure action profile is represented by a vector $a = (a_1, \dots, a_N)$, and the set of pure action profiles is denoted by $A \triangleq \prod_{i \in \mathcal{N}} A_i$. A mixed action for user i is a probability distribution over A_i and is denoted by $\alpha_i \in \Delta(A_i)$, where $\Delta(X)$ is the set of all probability distributions over a set X. A mixed action profile is represented by a vector $\alpha = (\alpha_1, \dots, \alpha_N) \in \prod_{i \in \mathcal{N}} \Delta(A_i)$. A mixed action profile of the users other than user i is written as $\alpha_{-i} = (\alpha_1, \dots, \alpha_{i-1}, \alpha_{i+1}, \dots, \alpha_N)$ so that α can be expressed as $\alpha = (\alpha_i, \alpha_{-i})$. Once a pure action profile of the users is determined, a signal is realized from the set of all possible signals, denoted Y, and is observed by the intervention device. We represent the probability distribution of signals by a mapping $\rho : A \to \Delta(Y)$. That is, $\rho(a) \in \Delta(Y)$ denotes the probability distribution of signals given a pure action profile a. When Y is finite, the probability that a signal y is realized given a pure action profile a is denoted by $\rho(y|a)$. After observing the realized signal, the intervention device takes its action, called an intervention action. We use a_0, α_0, and A_0 to denote a pure action, a mixed action, and the set of pure actions for the intervention device, respectively.

Since the intervention device chooses its action after observing the signal, a strategy for it can be represented by a mapping $f : Y \to \Delta(A_0)$, which is called an *intervention rule*. That is, $f(y) \in \Delta(A_0)$ denotes the mixed action for the intervention device when it observes a signal y. When A_0 is finite, the probability that the intervention device takes an action a_0 given a signal y is denoted by $f(a_0|y)$. The set of all possible intervention rules is denoted by $\mathcal{F}$. There is a system manager who determines the intervention rule used by the intervention device. We assume that the manager can commit to an intervention rule, for example, by using a protocol embedded in the intervention device. The payoffs of the users and the manager are determined by the actions of the intervention device and the users and the realized signal. We denote the payoff function of user $i \in \mathcal{N}$ by $u_i : A_0 \times A \times Y \to \mathbb{R}$ and that of the manager by $u_0 : A_0 \times A \times Y \to \mathbb{R}$. We call the pair

(Y,ρ) the *monitoring technology* of the intervention device, and call A_0 its *intervention capability*. An intervention device is characterized by these two, and we represent an *intervention scheme* by $\langle (Y,\rho), A_0, f\rangle$.

The game played by the manager and the users is formulated as an *intervention game*, which is summarized by the data

$$\Gamma = \langle \mathcal{N}_0, (A_i)_{i\in\mathcal{N}_0}, (u_i)_{i\in\mathcal{N}_0}, (Y,\rho)\rangle ,$$

where $\mathcal{N}_0 \triangleq \mathcal{N} \cup \{0\}$. The sequence of events in an intervention game can be listed as follows.

1) The manager chooses an intervention rule $f \in \mathcal{F}$.
2) The users choose their actions $\alpha \in \prod_{i\in\mathcal{N}} \Delta(A_i)$ simultaneously, knowing the intervention rule f chosen by the manager.
3) A pure action profile a is realized following the probability distribution α, and a signal $y \in Y$ is realized following the probability distribution $\rho(a)$.
4) The intervention device chooses its action $a_0 \in A_0$ following the probability distribution $f(y)$.

Ex ante payoffs, or expected payoffs given an intervention rule and a pure action profile, can be computed by taking expectations with respect to signals and intervention actions. The ex ante payoff function of user i is denoted by a function $v_i : \mathcal{F} \times A \to \mathbb{R}$, while that of the manager is denoted by $v_0 : \mathcal{F} \times A \to \mathbb{R}$. We say that an intervention game is *finite* if A_i, for $i \in \mathcal{N}_0$, and Y are all finite. In a finite intervention game, ex ante payoffs can be computed as

$$v_i(f,a) = \sum_{y\in Y}\sum_{a_0\in A_0} u_i(a_0,a,y) f(a_0|y)\rho(y|a),$$

for all $i \in \mathcal{N}_0$. Once the manager chooses an intervention rule f, the users play a simultaneous game, whose normal form representation is given by

$$\Gamma_f = \langle \mathcal{N}, (A_i)_{i\in\mathcal{N}}, (v_i(f,\cdot))_{i\in\mathcal{N}}\rangle .$$

We predict actions chosen by the users given an intervention rule f by applying the solution concept of Nash equilibrium [11] to the induced game Γ_f. With an abuse of notation, we extend the domain of v_i to $\mathcal{F} \times \prod_{i\in\mathcal{N}} \Delta(A_i)$ for all $i \in \mathcal{N}_0$ by taking expectation with respect to pure action profiles.

Definition 1: An intervention rule $f \in \mathcal{F}$ *sustains* an action profile $\alpha^* \in \prod_{i\in\mathcal{N}} \Delta(A_i)$ if α^* is a Nash equilibrium of the game Γ_f, i.e.,

$$v_i(f,\alpha_i^*,\alpha_{-i}^*) \geq v_i(f,\alpha_i,\alpha_{-i}^*)$$

for all $\alpha_i \in \Delta(A_i)$, for all $i \in \mathcal{N}$. An action profile α^* is *sustainable* if there exists an intervention rule f that sustains α^*.

Let $\mathcal{E}(f) \subseteq \prod_{i\in\mathcal{N}} \Delta(A_i)$ be the set of action profiles sustained by f. We say that a pair (f,α) is attainable if $\alpha \in \mathcal{E}(f)$. The manager's problem is to find an attainable pair that maximizes his ex ante payoff among all attainable pairs, which leads to the following solution concept for intervention games.

Definition 2: $(f^*,\alpha^*) \in \mathcal{F} \times \prod_{i\in\mathcal{N}} \Delta(A_i)$ is an *intervention equilibrium* if $\alpha^* \in \mathcal{E}(f^*)$ and

$$v_0(f^*,\alpha^*) \geq v_0(f,\alpha) \quad \text{for all } (f,\alpha) \text{ such that } \alpha \in \mathcal{E}(f).$$

$f^* \in \mathcal{F}$ is an *optimal intervention rule* if there exists an action profile $\alpha^* \in \prod_{i\in\mathcal{N}} \Delta(A_i)$ such that (f^*,α^*) is an intervention equilibrium.

An intervention equilibrium solves the following optimization problem:

$$\max_{(f,\alpha)} v_0(f,\alpha) \text{ subject to } \alpha \in \mathcal{E}(f). \quad (1)$$

The constraint $\alpha \in \mathcal{E}(f)$ represents incentive constraints for the users, which require that the users choose the action profile α in their self-interest given the intervention rule f. The problem (1) can be rewritten as $\max_{f\in\mathcal{F}} \max_{\alpha\in\mathcal{E}(f)} v_0(f,\alpha)$. Then an intervention equilibrium can be considered as a subgame perfect equilibrium (or Stackelberg equilibrium), with an implicit assumption that the manager can induce the users to choose the best Nash equilibrium for him in case of multiple Nash equilibria. Our interpretation is that, in order to achieve an intervention equilibrium (f^*,α^*), the manager announces the intervention rule f^* and recommends the action profile α^* to the users. Since $\alpha^* \in \mathcal{E}(f^*)$, the users do not have an incentive to deviate unilaterally from α^*, and α^* becomes a focal point [11] of the game Γ_{f^*}. Below we show the existence of an intervention equilibrium in a finite intervention game.

Proposition 1: Every finite intervention game has an intervention equilibrium.

We prove Proposition 1 using the following two lemmas.

Lemma 1: The correspondence $\mathcal{E} : \mathcal{F} \rightrightarrows \prod_{i\in\mathcal{N}} \Delta(A_i)$ is nonempty, compact-valued, and upper hemi-continuous.

Proof: We can show that, for any $f \in \mathcal{F}$, the set $\mathcal{E}(f)$ is nonempty by applying Nash Theorem [12] to Γ_f. Since $\prod_{i\in\mathcal{N}} \Delta(A_i)$ is bounded, it suffices to show that $\mathcal{E}$ has a closed graph to prove that $\mathcal{E}$ is compact-valued and upper hemi-continuous (u.h.c.) (see Theorem 3.4 of [13]). Choose a sequence $\{(f^n,\alpha^n)\}$ with $(f^n,\alpha^n) \to (f,\alpha)$ and $\alpha^n \in \mathcal{E}(f^n)$ for all n. Suppose that $\alpha \notin \mathcal{E}(f)$. Then there exists $i \in \mathcal{N}$ such that α_i is not a best response to α_{-i} in Γ_f. Then there exist $\epsilon > 0$ and α_i' such that $v_i(f,\alpha_i',\alpha_{-i}) > v_i(f,\alpha_i,\alpha_{-i}) + 3\epsilon$. Since v_i is continuous and $(f^n,\alpha^n) \to (f,\alpha)$, for sufficiently large n we have

$$\begin{aligned} v_i(f^n,\alpha_i',\alpha_{-i}^n) &> v_i(f,\alpha_i',\alpha_{-i}) - \epsilon \\ &> v_i(f,\alpha_i,\alpha_{-i}) + 2\epsilon > v_i(f^n,\alpha_i^n,\alpha_{-i}^n) + \epsilon, \end{aligned}$$

which contradicts $\alpha^n \in \mathcal{E}(f^n)$. ■

Define a function $\check{v}_0 : \mathcal{F} \to \mathbb{R}$ by $\check{v}_0(f) = \max_{\alpha\in\mathcal{E}(f)} v_0(f,\alpha)$. For each f, $\mathcal{E}(f)$ is nonempty and compact by Lemma 1 and $v_0(f,\cdot)$ is continuous. Hence, the function $\check{v}_0$ is well-defined.

Lemma 2: The function $\check{v}_0$ is upper semi-continuous.

Proof: Let $E(f) = \{\alpha \in \mathcal{E}(f) : v_0(f,\alpha) = \check{v}_0(f)\}$. Note that $E(f)$ is nonempty for all f. Fix f, and let $\{f^n\}$ be any sequence converging to f. Choose $\alpha^n \in E(f^n)$, for all n. Let $v_0^s = \limsup_{n\to\infty} \check{v}_0(f^n)$. Then there exists a subsequence $\{f^{n_k}\}$ such that $v_0^s = \lim v_0(f^{n_k},\alpha^{n_k})$. Since $\alpha^n \in \mathcal{E}(f^n)$ and $\mathcal{E}$ is u.h.c., there exists a convergent subsequence of $\{\alpha^{n_k}\}$, called $\{\alpha^j\}$, whose limit point α is in $\mathcal{E}(f)$. Hence, $v_0^s = \lim v_0(f^j,\alpha^j) = v_0(f,\alpha) \leq \check{v}_0(f)$ since $\alpha \in \mathcal{E}(f)$. ■

Note that, in a finite intervention game, the space of intervention rules, $\mathcal{F}$, is equivalent to $(\Delta(A_0))^{|Y|}$, which is compact. Therefore, a solution to $\max_{f\in\mathcal{F}} \check{v}_0(f)$ exists, which establishes the existence of an intervention equilibrium. This completes the proof of Proposition 1.

There can be multiple intervention equilibria, all of which yield the same payoff for the manager. We can propose different selection criteria for the manager to choose an intervention equilibrium out of multiple ones. For example, the discussion on affine intervention rules in Section IV-A is motivated by the robustness of performance to mistakes by the users as well as simplicity.

Recall that an intervention device is characterized by (Y, ρ) and A_0. In this paper, we focus on the problem of finding an optimal intervention rule when the manager has a particular intervention device. However, we can think of a scenario where the manager can select an intervention device from multiple ones given the operating cost of each available intervention device. Our analysis in this paper allows the manager to evaluate the optimal performance achieved with each intervention device. He can then select the best intervention device taking into account both performance and cost.

III. Performance with Intervention under Imperfect Monitoring

A. Analytical Results

In this section, we maintain the following assumption.

Assumption 1: There exists an action for the intervention device $\tilde{a}_0 \in A_0$ that satisfies

$$u_0(\tilde{a}_0, a, y) > u_0(a_0, a, y) \quad \text{for all } a_0 \neq \tilde{a}_0,$$

for all $a \in A$ and $y \in Y$.

Assumption 1 asserts the existence of an intervention action that is most preferred by the manager regardless of the action profile of the users and the signal. We can interpret the most preferred intervention action, $\tilde{a}_0$, as the intervention action that corresponds to no intervention. Then Assumption 1 states that exerting intervention is costly for the manager, reflecting that intervention typically degrades the overall performance. Moreover, there is some operational cost (e.g., energy consumption) needed to exert intervention.

Define an intervention rule $\tilde{f}$ by $\tilde{f}(y) = \tilde{a}_0$ for all y. It can be considered that the manager decides not to intervene at all when he chooses $\tilde{f}$. Let $\overline{v}_0 = \sup_{(f,\alpha)} v_0(f, \alpha)$, $v_0^* = \sup_f \sup_{\alpha\in\mathcal{E}(f)} v_0(f, \alpha)$, and $\tilde{v}_0 = \sup_{\alpha\in\mathcal{E}(\tilde{f})} v_0(\tilde{f}, \alpha)$. $\overline{v}_0$ is the best performance that the manager can obtain when the users are not subject to the incentive constraints (e.g., when the actions of the users can be completely controlled by the manager). v_0^* is the best performance when the manager is required to satisfy the incentive constraints for the users. Lastly, $\tilde{v}_0$ is the best performance when the manager does not engage in active intervention. It is straightforward to see that $\tilde{v}_0 \leq v_0^* \leq \overline{v}_0$. The following proposition provides a sufficient condition on the intervention game for a gap between $\overline{v}_0$ and v_0^* to exist.

Proposition 2: Suppose that the intervention game is finite, ρ has full support (i.e., $\rho(y|a) > 0$ for all y and a), and there is no α such that $\alpha \in \mathcal{E}(\tilde{f})$ and $v_0(\tilde{f}, \alpha) = \overline{v}_0$. Then $v_0^* < \overline{v}_0$.

Proof: Suppose that the conclusion does not hold, i.e., $v_0^* = \overline{v}_0$. Since the intervention game is finite, v_0^* is attained by Proposition 1. Thus, there exists (f^*, α^*) such that $\alpha^* \in \mathcal{E}(f^*)$ and $v_0(f^*, \alpha^*) = \overline{v}_0$. Note that $\overline{v}_0 = v_0(f^*, \alpha^*) \leq v_0(\tilde{f}, \alpha^*) \leq \overline{v}_0$. Hence, $v_0(f^*, \alpha^*) = v_0(\tilde{f}, \alpha^*)$. Since ρ has full support, we have $f^*(y) = \tilde{f}(y)$ for all y. This contradicts the hypothesis that there is no α such that $\alpha \in \mathcal{E}(\tilde{f})$ and $v_0(\tilde{f}, \alpha) = \overline{v}_0$. ■

When the intervention game is finite, $\overline{v}_0$ is attained since v_0 is continuous and $(\mathcal{F} \times \prod_{i\in\mathcal{N}} \Delta(A_i))$ is compact. Since $v_0(\tilde{f}, \alpha) \geq v_0(f, \alpha)$ for all α, for all f, we have $\overline{v}_0 = \max_\alpha v_0(\tilde{f}, \alpha)$. In fact, when the intervention game is finite and ρ has full support, $\tilde{f}$ is the only intervention rule that can attain the best feasible performance, $\overline{v}_0$. When $\tilde{f}$ sustains no action profile that attains $\overline{v}_0$, the manager needs to trigger a punishment following some signals in order to provide appropriate incentives for the users to follow an action profile such that $v_0(\tilde{f}, \alpha) = \overline{v}_0$. However, since ρ has full support, the punishment results in a performance loss, which prevents the manager from achieving $\overline{v}_0$.

B. Illustrative Example (Type-2 Intervention)

We consider a wireless network where two users interfere with each other. Each user has two pure actions, a_L and a_H, which represent low and high resource usage levels, respectively, and satisfy $0 < a_L < a_H$. The service quality is determined randomly given an action profile, and there are two possible quality levels, $\overline{y}$ and $\underline{y}$, with $0 < \underline{y} < \overline{y}$. The service quality is realized following the distribution

$$\rho(\overline{y}|a) = \begin{cases} p, & \text{if } a = (a_L, a_L), \\ q, & \text{if } a = (a_H, a_L) \text{ or } (a_L, a_H), \\ r, & \text{if } a = (a_H, a_H), \end{cases}$$

where $0 < r < q < p < 1$. The intervention device in this example acts as a gatekeeper (i.e., type-2 intervention) after observing the service quality, having two pure actions: intervene ($\hat{a}_0$) and not intervene ($\tilde{a}_0$). When the intervention device does not intervene, a user receives a payoff given by the product of the quality level and its own usage level, i.e., $u_i(\tilde{a}_0, a, y) = y a_i$ for all a and y, for $i = 1, 2$. When the intervention device does intervene, the service stops completely and a user receives zero payoff regardless of its usage level, i.e., $u_i(\hat{a}_0, a, y) = 0$ for all a and y, for $i = 1, 2$. The payoff of the manager is set as the average payoff of the users, i.e., $u_0(a_0, a, y) = [u_1(a_0, a, y) + u_2(a_0, a, y)]/2$. Note that denoting the action of not intervening by $\tilde{a}_0$ is consistent with Assumption 1. A communication scenario that fits into this example is presented in Fig. 3.

Since there are only two pure actions for the intervention device, we can represent $\mathcal{F} = [0, 1]$ and use $f(y)$ as the probability of not intervening given the signal y. The ex ante payoff function of user i is given by

$$v_i(f, a) = [\rho(\overline{y}|a) f(\overline{y})\overline{y} + (1 - \rho(\overline{y}|a)) f(\underline{y})\underline{y}] a_i.$$

The payoff matrix of the game $\Gamma_{\tilde{f}}$, i.e., the game when the intervention device does not intervene at all, is displayed in Table I, where we define $y_k = k\overline{y} + (1 - k)\underline{y}$, for $k = p, q, r$. We assume that the game $\Gamma_{\tilde{f}}$ is the prisoner's dilemma game, i.e., $y_q a_H > y_p a_L > y_r a_H > y_q a_L$ and $2y_p a_L > y_q(a_H + a_L)$. Then without any intervention, it is the

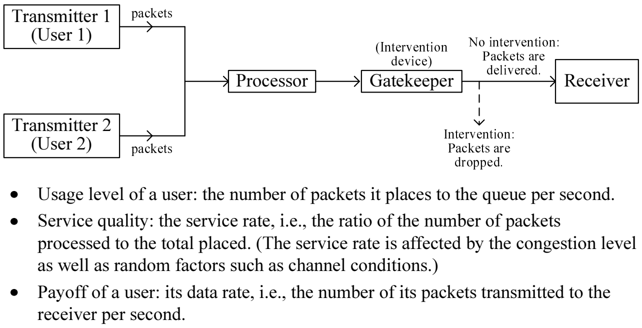


Fig. 3. A communication scenario that fits into the example in Section III-B.

dominant strategy of each user to choose the high usage level, which results in the inefficient Nash equilibrium. The manager aims to improve the inefficiency of the Nash equilibrium by providing appropriate incentives through intervention.[1] We restrict attention to symmetric action profiles, assuming that the manager desires to sustain a symmetric action profile.

Let $w_0(\alpha) = \sup_f \{v_0(f,\alpha) : \alpha \in \mathcal{E}(f)\}$. That is, $w_0(\alpha)$ is the maximum payoff that the manager can obtain while sustaining a given action profile α. Since we focus on symmetric action profiles and there are only two pure actions for each user, let $\alpha \in [0,1]$ denote the probability of each user playing a_L. Then we can show that $w_0(0) = y_r a_H$ and, for $\alpha \in (0,1]$,

$$w_0(\alpha) = \begin{cases} \frac{\{(q-r)+\alpha[(p-q)-(q-r)]\}a_H a_L}{[(1-r)a_H-(1-q)a_L]+\alpha[(pa_L-qa_H)-(qa_L-ra_H)]}\overline{y}, \\ \quad \text{if } \alpha(pa_L-qa_H)+(1-\alpha)(qa_L-ra_H) \geq 0, \\ 0, \quad \text{otherwise.} \end{cases} \tag{2}$$

The intervention rule that attains $w_0(0)$ is given by $\tilde{f}$ (i.e., no intervention), while the intervention rule that attains $w_0(\alpha)$, for $\alpha \in (0,1]$, is given by $f(\overline{y}) = 1$ and

$$f(\underline{y}) = \frac{(qa_L-ra_H)+\alpha[(pa_L-qa_H)-(qa_L-ra_H)]}{[(1-r)a_H-(1-q)a_L]+\alpha[(pa_L-qa_H)-(qa_L-ra_H)]}\frac{\overline{y}}{\underline{y}}$$

if $\alpha(pa_L - qa_H) + (1-\alpha)(qa_L - ra_H) \geq 0$, and by $f(\overline{y}) = f(\underline{y}) = 0$ otherwise. We can think of $\alpha(pa_L - qa_H) + (1-\alpha)(qa_L - ra_H)$ as a measure of the sensitivity of signals between the two pure actions when the other user plays α. When signals are sufficiently sensitive at α, an intervention rule can sustain α with a positive payoff by degrading the low quality only. On the contrary, when signals are not sensitive, destroying all the payoffs is the only method to sustain α, which yields zero payoff for the users. Note that, when $pa_L - qa_H < 0$ and $qa_L - ra_H > 0$, the pure action profile (a_L, a_L) cannot be sustained with a positive payoff while a completely mixed action profile can be. In this case, signals are more sensitive to the action of a user when the other user plays a_H. Hence, by inducing the users to play a_H with positive probability, the manager can make the signal a more informative indicator of a deviation. This allows the possibility that an intervention rule improves the performance of non-cooperative equilibrium by sustaining a completely mixed action profile even when the social optimum (a_L, a_L) cannot be sustained non-trivially. A similar discussion about the advantage of using mixed actions can be found in [14] in the context of the repeated prisoner's dilemma game.

[1]In this paper, we focus on the role of intervention schemes to improve the prospect of cooperation by applying intervention to prisoner's dilemma situations. Intervention schemes can also be used to help users achieve coordination by eliminating the multiplicity of Nash equilibria in coordination games such as the battle of the sexes and the stag hunt [11]. For example, in the stag-hunt game, an intervention scheme may induce players to choose the payoff dominant (but not risk dominant) "all stag" equilibrium by intervening in the hare hunt.

TABLE I
PAYOFF MATRIX OF THE GAME $\Gamma_{\tilde{f}}$ IN THE ILLUSTRATIVE EXAMPLE IN SECTION III-B.

	a_L	a_H
a_L	$y_p a_L,\ y_p a_L$	$y_q a_L,\ y_q a_H$
a_H	$y_q a_H,\ y_q a_L$	$y_r a_H,\ y_r a_H$

In this example, we have $\tilde{v}_0 = w_0(0) = y_r a_H$, $\overline{v}_0 = y_p a_L$, and $v_0^* = \max_{\alpha \in [0,1]} w_0(\alpha)$. We summarize the results about the performance with intervention, v_0^*, in the following proposition.

Proposition 3: (i) Suppose that (a) $pa_L - qa_H < 0$ and $qa_L - ra_H < 0$, or (b) $pa_L - qa_H < qa_L - ra_H$ and $(p-q)(1-r) - (q-r)(1-q) \leq 0$. Then $v_0^* = \tilde{v}_0$.
(ii) Suppose that (c) $pa_L - qa_H \geq qa_L - ra_H \geq 0$, (d) $pa_L - qa_H \geq 0 > qa_L - ra_H$, or (e) $0 \leq pa_L - qa_H < qa_L - ra_H$ and $(p-q)(1-r)-(q-r)(1-q) > 0$. Then $v_0^* = \max\{\tilde{v}_0, w_0(1)\}$.
(iii) Suppose that (f) $pa_L - qa_H < 0 \leq qa_L - ra_H$ and $(p-q)(1-r)-(q-r)(1-q) > 0$. Then $v_0^* = \max\{\tilde{v}_0, w_0(\overline{\alpha})\}$, where

$$\overline{\alpha} = \frac{qa_L - ra_H}{(qa_L - ra_H) - (pa_L - qa_H)}.$$

Proof: See Appendix A. ■

Fig. 4 shows that each of the three cases of $v_0^* = \tilde{v}_0$, $v_0^* = w_0(\overline{\alpha})$, and $v_0^* = w_0(1)$ can arise depending on the parameter values. To obtain the results, we set $a_L = 1$, $a_H = 1.19$, $\overline{y} = 5$, $\underline{y} = 1$, $q = 0.8$, and $r = 0.65$ while varying $p = 0.9, 0.94, 0.96$. We can see that, as p increases, the performance with intervention improves, getting closer to its upper bound $\overline{v}_0$. In fact, when $v_0^* = w_0(1)$, we have

$$\overline{v}_0 - v_0^* = \frac{(1-p)a_L(y_q a_H - y_p a_L)}{(1-q)a_H - (1-p)a_L} > 0,$$

which is consistent with Proposition 2. The gap between v_0^* and $\overline{v}_0$ vanishes as p approaches 1, while it increases with the deviation gain $(y_q a_H - y_p a_L)$. This result is intuitive because punishment rarely occurs when p is close to 1 while a stronger punishment is needed as the deviation gain is larger.

We can consider pricing schemes applied to this example, by having the manager charge different payments depending on the realized service quality. In order to find a pricing scheme that sustains a certain action profile, the manager needs to know how payments affect the payoffs of the users (i.e., the function $u_i(a_0, a, y)$, where a_0 is now interpreted as the charged payments). Suppose, for example, that the payoff of each user from resource usage is given by its data rates. Since intervention influences data rates directly, it is relatively easy to find out how intervention actions affect payoffs. In contrast, finding out how payments affect payoffs requires the manager to know how the users value payments relative to data rates. This information is difficult to obtain since the

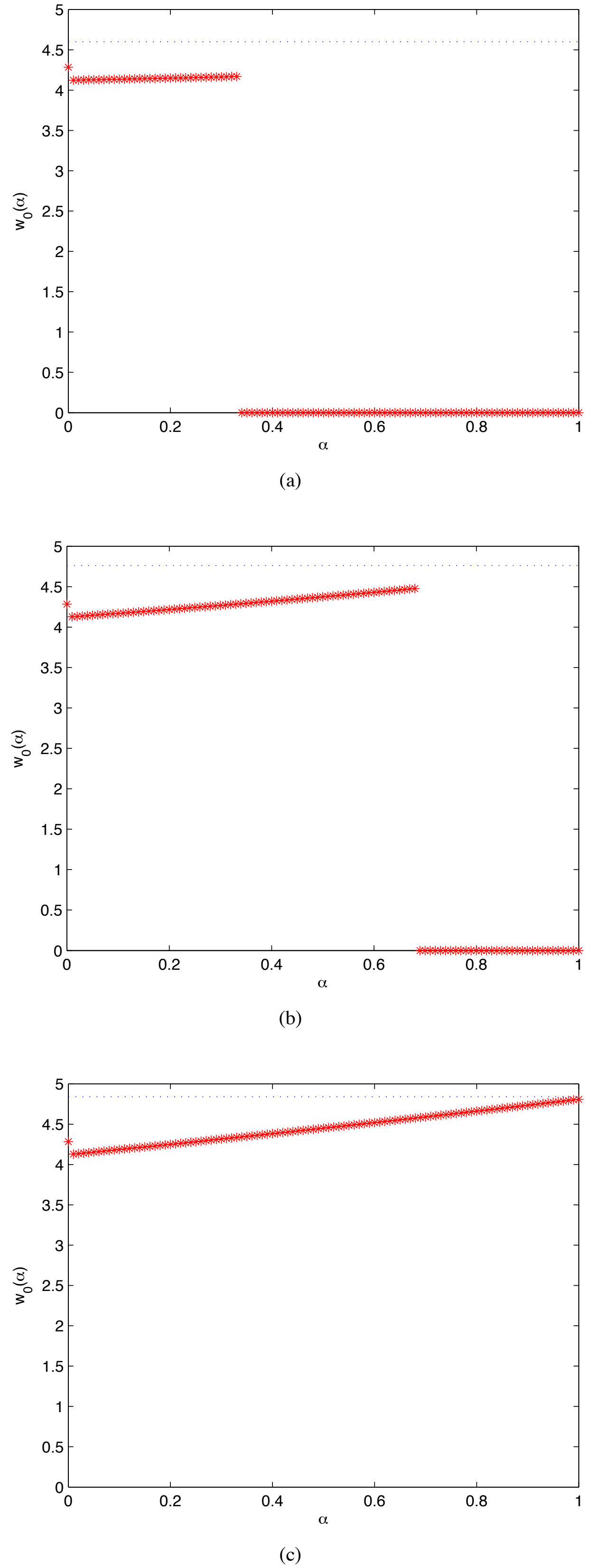


Fig. 4. The graph of the function $w_0(\alpha)$ defined in (2): (a) $v_0^* = \tilde{v}_0$ ($p = 0.9$), (b) $v_0^* = w_0(\overline{\alpha})$ ($p = 0.94$), and (c) $v_0^* = w_0(1)$ ($p = 0.96$). (The dotted lines display $\overline{v}_0 = y_p a_L$.)

users' valuations are subjective and thus not easily measurable. This discussion points out the informational advantage of intervention over pricing.

IV. Performance with Intervention under Perfect Monitoring

A. Analytical Results

In this section, we consider the case where the intervention device can observe the pure action profile without errors (i.e., perfect monitoring), as stated formally in the following assumption.

Assumption 2: $Y = A$, and only signal a can arise in the distribution $\rho(a)$ for all $a \in A$.

With Assumption 2, we always have $y = a$, and thus we write the payoff functions more compactly as $u_i(a_0, a)$ instead of $u_i(a_0, a, a)$, for all $i \in \mathcal{N}_0$. We also maintain the following two assumptions in this section.

Assumption 3: There exists an action for the intervention device $\underline{a}_0 \in A_0$ that satisfies, for all $i \in \mathcal{N}_0$,

$$u_i(\underline{a}_0, a) \geq u_i(a_0, a) \quad \text{for all } a_0 \in A_0, \text{ for all } a \in A. \quad (3)$$

Assumption 4: A_0 is compact, and $u_i : A_0 \times A \to \mathbb{R}$ is continuous for all $i \in \mathcal{N}_0$.

Assumption 3 states that there exists an intervention action that is most preferred by the users and the manager regardless of the action profile of the users. We can interpret $\underline{a}_0$ in Assumption 3 as the intervention action corresponding to no intervention, similarly to $\tilde{a}_0$ in Assumption 1. Then Assumption 3 implies that intervention can only reduce the payoffs of the users and the manager.

In this section, we restrict attention to pure actions (both for the users and for the intervention device) while allowing the action spaces to be continuous spaces. Thus, an intervention rule is represented by a mapping $f : A \to A_0$, while user i chooses a pure action $a_i \in A_i$ given an intervention rule. Then the ex ante payoff function is given by $v_i(f, a) = u_i(f(a), a)$, for all $i \in \mathcal{N}_0$. We define a class of intervention rules.

Definition 3: $f_{\tilde{a}} : A \to A_0$ is an *extreme intervention rule with target action profile* $\tilde{a} \in A$ if $f_{\tilde{a}}$ satisfies

- $f_{\tilde{a}}(a) \in \arg\min_{a_0 \in A_0} u_i(a_0, a)$ if $\exists\, i \in \mathcal{N}$ such that $a_i \neq \tilde{a}_i$ and $a_j = \tilde{a}_j\ \forall j \neq i$, and
- $f_{\tilde{a}}(a) = \underline{a}_0$ otherwise.

By Assumption 4, $\arg\min_{a_0 \in A_0} u_i(a_0, a)$ is non-empty for all $a \in A$ and $i \in \mathcal{N}$. Thus, for every $\tilde{a} \in A$, there exists an extreme intervention rule with target action profile $\tilde{a}$. An extreme intervention rule prescribes an intervention action that minimizes the payoff of the deviator if there is a unilateral deviation from the target action profile while prescribing no intervention if there is no unilateral deviation. Hence, an extreme intervention rule provides the strongest incentive for the users to follow a given target action profile. Let $\mathcal{E}(\mathcal{F}) = \cup_{f \in \mathcal{F}} \mathcal{E}(f)$. That is, $\mathcal{E}(\mathcal{F})$ is the set of all sustainable action profiles.

Lemma 3: If $a^* \in \mathcal{E}(\mathcal{F})$, then $a^* \in \mathcal{E}(f_{a^*})$.

Proof: Suppose that $a^* \in \mathcal{E}(\mathcal{F})$. Then there exists an intervention rule f such that $v_i(f, a^*) \geq v_i(f, a_i, a_{-i}^*)$ for all $a_i \in A_i$, for all $i \in \mathcal{N}$. Then we obtain $v_i(f_{a^*}, a^*) = u_i(\underline{a}_0, a^*) \geq u_i(f(a^*), a^*) \geq u_i(f(a_i, a_{-i}^*), a_i, a_{-i}^*) \geq u_i(f_{a^*}(a_i, a_{-i}^*), a_i, a_{-i}^*) = v_i(f_{a^*}, a_i, a_{-i}^*)$ for all $a_i \neq a_i^*$, for all $i \in \mathcal{N}$, where the first inequality follows from (3) and the third from the definition of extreme intervention rules. ■

Let $\mathcal{E}^* = \{a \in A : a \in \mathcal{E}(f_a)\}$. The following results are the consequences of Lemma 3.

Proposition 4: (i) $\mathcal{E}(\mathcal{F}) = \mathcal{E}^*$.
(ii) If (f^*, a^*) is an intervention equilibrium, then (f_{a^*}, a^*) is also an intervention equilibrium.

Proof: (i) Let $a^* \in \mathcal{E}^*$. Then $a^* \in \mathcal{E}(f_{a^*}) \subset \mathcal{E}(\mathcal{F})$. Hence, $\mathcal{E}^* \subset \mathcal{E}(\mathcal{F})$. The other inclusion $\mathcal{E}(\mathcal{F}) \subset \mathcal{E}^*$ follows from Lemma 3.

(ii) Suppose that (f^*, a^*) is an intervention equilibrium. Then by Definition 2, $a^* \in \mathcal{E}(f^*)$ and $v_0(f^*, a^*) \geq v_0(f, a)$ for all $(f, a) \in \mathcal{F} \times A$ such that $a \in \mathcal{E}(f)$. Since $a^* \in \mathcal{E}(\mathcal{F})$, $a^* \in \mathcal{E}(f_{a^*})$ by Lemma 3. Hence, $v_0(f^*, a^*) \geq v_0(f_{a^*}, a^*)$. On the other hand, since $f_{a^*}(a^*) = \underline{a}_0$, we have $v_0(f^*, a^*) \leq v_0(f_{a^*}, a^*)$ by (3). Therefore, $v_0(f^*, a^*) = v_0(f_{a^*}, a^*)$, and thus $v_0(f_{a^*}, a^*) \geq v_0(f, a)$ for all $(f, a) \in \mathcal{F} \times A$ such that $a \in \mathcal{E}(f)$. This proves that (f_{a^*}, a^*) is an intervention equilibrium. ■

Proposition 4 shows that it is without loss of generality to restrict attention to pairs of the form (f_a, a) when we ask whether a given action profile is sustainable and whether there exists an intervention equilibrium. The basic idea is that, in order to sustain an action profile, it suffices to consider an intervention rule that punishes a deviator most severely. The role of extreme intervention rules is analogous to that of optimal penal codes [15] in repeated games with perfect monitoring. The following proposition characterizes intervention equilibria among pairs of the form (f_a, a).

Proposition 5: (f_{a^*}, a^*) is an intervention equilibrium if and only if $a^* \in \mathcal{E}^*$ and $u_0(\underline{a}_0, a^*) \geq u_0(\underline{a}_0, a)$ for all $a \in \mathcal{E}^*$.

Proof: Suppose that (f_{a^*}, a^*) is an intervention equilibrium. Then $a^* \in \mathcal{E}(f_{a^*})$, and thus $a^* \in \mathcal{E}^*$. Also, $v_0(f_{a^*}, a^*) \geq v_0(f, a)$ for all (f, a) such that $a \in \mathcal{E}(f)$. Choose any $a \in \mathcal{E}^*$. Then $a \in \mathcal{E}(f_a)$, and thus $u_0(\underline{a}_0, a^*) = v_0(f_{a^*}, a^*) \geq v_0(f_a, a) = u_0(\underline{a}_0, a)$.

Suppose that $a^* \in \mathcal{E}^*$ and $u_0(\underline{a}_0, a^*) \geq u_0(\underline{a}_0, a)$ for all $a \in \mathcal{E}^*$. To prove that (f_{a^*}, a^*) is an intervention equilibrium, we need to show that (i) $a^* \in \mathcal{E}(f_{a^*})$, and (ii) $v_0(f_{a^*}, a^*) \geq v_0(f, a)$ for all (f, a) such that $a \in \mathcal{E}(f)$. (i) follows from $a^* \in \mathcal{E}^*$. To prove (ii), choose any (f, a) such that $a \in \mathcal{E}(f)$. By Lemma 3, we have $a \in \mathcal{E}^*$. Then $v_0(f_{a^*}, a^*) = u_0(\underline{a}_0, a^*) \geq u_0(\underline{a}_0, a) \geq v_0(f, a)$, where the first inequality follows from $a \in \mathcal{E}^*$. ■

Proposition 5 shows that the pair (f_a, a) constitutes an intervention equilibrium if a solves

$$\max_{a \in \mathcal{E}^*} u_0(\underline{a}_0, a). \tag{4}$$

The next proposition provides a sufficient condition under which an intervention equilibrium exists.

Proposition 6: If A_i is a bounded set in Euclidean space for all $i \in \mathcal{N}$, then there exists an intervention equilibrium.

Proof: By Proposition 4(ii) and Proposition 5, an intervention equilibrium exists if and only if there exists a solution to the problem (4). Since $u_0(\underline{a}_0, a)$ is continuous in a, the result follows if we show that the constraint set $\mathcal{E}^*$ is compact. Since $\mathcal{E}^* \subset A$ and A is bounded, $\mathcal{E}^*$ is also bounded. Let $G_i(a) \triangleq \arg\min_{a_0 \in A_0} u_i(a_0, a)$ for all $a \in A$, for all $i \in \mathcal{N}$. By the Theorem of the Maximum [13], $G_i(a)$ is compact-valued and u.h.c. To show that $\mathcal{E}^*$ is closed, choose a sequence $\{a^n\}$ with $a^n \to a^*$ and $a^n \in \mathcal{E}^*$ for all n. Choose any $i \in \mathcal{N}$ and $a'_i \in A_i$. Let $\{a_0^n\}$ be a sequence such that $a_0^n \in G_i(a'_i, a_{-i}^n)$ for all n. Since $a^n \in \mathcal{E}(f_{a^n})$, we have $u_i(\underline{a}_0, a^n) \geq u_i(a_0^n, a'_i, a_{-i}^n)$. Also, since $G_i(a)$ is u.h.c., there exists a convergent subsequence of $\{a_0^n\}$ whose limit point a_0^* is in $G_i(a'_i, a_{-i}^*)$. Since u_i is continuous, we obtain $u_i(\underline{a}_0, a^*) \geq u_i(a_0^*, a'_i, a_{-i}^*)$ by taking limits. This proves $a^* \in \mathcal{E}(f_{a^*})$ and thus $a^* \in \mathcal{E}^*$. ■

Now we turn to the question of whether the best feasible performance, $\overline{v}_0$, can be achieved with intervention. At an intervention equilibrium of the form (f_{a^*}, a^*), intervention exists only as a threat to deter deviation, and no intervention is exerted as long as the users follow the target action profile. This contrasts with the imperfect monitoring scenario considered in Proposition 2, where providing incentives requires that intervention be used sometimes even when the users follow the target action profile, which results in a performance loss. Thus, with perfect monitoring, it is possible for an intervention scheme to achieve the best feasible performance as long as the intervention capability is sufficiently strong. This discussion is formally stated below as a corollary of Proposition 5. Note that $\overline{v}_0 = \sup_{a \in A} u_0(\underline{a}_0, a)$, which is attained when A is compact.

Corollary 1: If $a^o \in \arg\max_{a \in A} u_0(\underline{a}_0, a)$ and $u_i(\underline{a}_0, a^o) \geq u_i(f_{a^o}(a_i, a_{-i}^o), a_i, a_{-i}^o)$ for all $a_i \in A_i$, for all $i \in \mathcal{N}$, then $v_0^* = \overline{v}_0$.

Extreme intervention rules are useful to characterize sustainable action profiles and intervention equilibria. However, they may not be desirable in practice. For example, when a user chooses an action different from the target action by mistake (i.e., trembling hands), an extreme intervention rule triggers the most severe punishment for the user, which may result in a large performance loss. Thus, it is of interest to investigate intervention rules that use weaker punishments than extreme intervention rules do. To obtain concrete results, we assume that $A_i = [\underline{a}_i, \overline{a}_i] \subset \mathbb{R}$ with $\underline{a}_i < \overline{a}_i$ for all $i \in \mathcal{N}_0$ in the remainder of this subsection. Below we define another class of intervention rules.

Definition 4: $f_{\tilde{a},c} : A \to A_0$ is a *(truncated) affine intervention rule with target action profile* $\tilde{a} \in A$ *and intervention rate profile* $c \in \mathbb{R}^N$ if

$$f_{\tilde{a},c}(a) = [c \cdot (a - \tilde{a}) + \underline{a}_0]_{\underline{a}_0}^{\overline{a}_0},$$

where $[x]_\alpha^\beta = \min\{\max\{x, \alpha\}, \beta\}$.

The following proposition constructs an affine intervention rule to sustain an interior target action profile in the differentiable payoff case.

Proposition 7: Let $a^* \in \mathcal{A}$ be an action profile such that $a_i^* \in (\underline{a}_i, \overline{a}_i)$ for all $i \in \mathcal{N}$. Suppose that, for all $i \in \mathcal{N}$, u_i is twice continuously differentiable and $u_i(a_0, a^*)$ is strictly decreasing in a_0 on $[\underline{a}_0, \overline{a}_0]$. Let

$$c_i^* = -\frac{\partial u_i(\underline{a}_0, a^*)/\partial a_i}{\partial u_i(\underline{a}_0, a^*)/\partial a_0} \tag{5}$$

for all $i \in \mathcal{N}$.[2] Suppose that

$$\frac{\partial^2 u_i}{\partial a_i^2}(\underline{a}_0, a_i, a_{-i}^*) \leq 0 \quad \text{for all } a_i \in (\underline{a}_i, \overline{a}_i)$$

[2]We define $\partial u_i(\underline{a}_0, a^*)/\partial a_0$ as the right partial derivative of u_i with respect to a_0 at $(\underline{a}_0, a^*)$.

for all $i \in \mathcal{N}$ such that $c_i^* = 0$,

$$\frac{\partial^2 u_i}{\partial a_i^2}(\underline{a}_0, a_i, a_{-i}^*) \leq 0 \quad \text{for all } a_i \in (\underline{a}_i, a_i^*),$$

$$(c_i^*)^2 \frac{\partial^2 u_i}{\partial a_0^2} + 2c_i^* \frac{\partial^2 u_i}{\partial a_i \partial a_0} + \frac{\partial^2 u_i}{\partial a_i^2} \leq 0$$

(left-hand side evaluated at $(c_i^*(a_i - a_i^*) + \underline{a}_0, a_i, a_{-i}^*)$) for all $a_i \in (a_i^*, \min\{\overline{a}_i, a_i^* + (\overline{a}_0 - \underline{a}_0)/c_i^*\})$, and

$$\frac{\partial u_i}{\partial a_i}(\overline{a}_0, a_i, a_{-i}^*) \leq 0 \quad \text{for all } a_i \in (a_i^* + (\overline{a}_0 - \underline{a}_0)/c_i^*, \overline{a}_i)$$

for all $i \in \mathcal{N}$ such that $c_i^* > 0$, and

$$\frac{\partial u_i}{\partial a_i}(\overline{a}_0, a_i, a_{-i}^*) \geq 0 \quad \text{for all } a_i \in (\underline{a}_i, a_i^* + (\overline{a}_0 - \underline{a}_0)/c_i^*),$$

$$(c_i^*)^2 \frac{\partial^2 u_i}{\partial a_0^2} + 2c_i^* \frac{\partial^2 u_i}{\partial a_i \partial a_0} + \frac{\partial^2 u_i}{\partial a_i^2} \leq 0$$

(left-hand side evaluated at $(c_i^*(a_i - a_i^*) + \underline{a}_0, a_i, a_{-i}^*)$) for all $a_i \in (\max\{\overline{a}_i, a_i^* + (\overline{a}_0 - \underline{a}_0)/c_i^*\}, a_i^*)$, and

$$\frac{\partial^2 u_i}{\partial a_i^2}(\underline{a}_0, a_i, a_{-i}^*) \leq 0 \quad \text{for all } a_i \in (a_i^*, \overline{a}_i)$$

for all $i \in \mathcal{N}$ such that $c_i^* < 0$.[3] Then f_{a^*,c^*} sustains a^*.

Proof: See the Appendix of [1]. ■

Note that $\partial u_i(\underline{a}_0, a^*)/\partial a_0 < 0$ for all $i \in \mathcal{N}$ since $u_i(a_0, a^*)$ is strictly decreasing in a_0. Thus, c_i^*, defined in (5), has the same sign as $\partial u_i(\underline{a}_0, a^*)/\partial a_i$. With $A_0 = [\underline{a}_0, \overline{a}_0]$, the intervention action can be interpreted as the intervention level, and at the target action profile a^* the users receive higher payoffs as the intervention level is smaller. The affine intervention rule f_{a^*,c^*}, constructed in Proposition 7, has the properties that the intervention device uses the minimum intervention level $\underline{a}_0$ when the users choose the target action profile a^*, i.e., $f_{a^*,c^*}(a^*) = \underline{a}_0$, and that the intervention level increases in the rate of $|c_i^*|$ as user i deviates to the direction in which its payoff increases at $(\underline{a}_0, a^*)$. The expression of c_i^* in (5) has an intuitive explanation. Since c_i^* is proportional to $\partial u_i(\underline{a}_0, a^*)/\partial a_i$ and inversely proportional to $-\partial u_i(\underline{a}_0, a^*)/\partial a_0$, a user faces a higher intervention rate as its incentive to deviate from $(\underline{a}_0, a^*)$ is stronger and as a change in the intervention level has a smaller impact on its payoff. The intervention level does not react to the action of user i when $c_i^* = 0$, because user i chooses a_i^* in its self-interest even when the intervention level is fixed at $\underline{a}_0$, provided that the other users choose a_{-i}^*. Finally, we note that if (f^*, a^*) is an intervention equilibrium and $f_{a^*,c}$ sustains a^* for some c, then $(f_{a^*,c}, a^*)$ is also an intervention equilibrium, since $f_{a^*,c}(a^*) = \underline{a}_0$.

[3] We define $(\alpha, \beta) = \varnothing$ if $\alpha \geq \beta$.

B. Illustrative Example (Type-1 Intervention)

As an illustrative example, we consider another resource sharing scenario in a wireless network where $N \geq 2$ users and an intervention device interfere with each other. In this example, the intervention device engages in type-1 intervention, affecting the service quality through its usage level. The actions of the users and the intervention device are their usage levels, and the action space is given by $A_i = [0, \overline{a}_i]$ for all $i \in \mathcal{N}_0$. $\overline{a}_i$ denotes the maximum usage level of user i, and $\overline{a}_0$ denotes that of the intervention device, which can be considered as its intervention capability. We assume that $\overline{a}_i \geq q/2b$ for all $i \in \mathcal{N}$, while imposing no restriction on $\overline{a}_0$. The service quality is determined by the total usage level, $a_0 + \sum_{i=1}^N a_i$, following the relationship

$$Q(a_0, a) = \left[q - b \left(a_0 + \sum_{i=1}^N a_i \right) \right]^+,$$

where $q, b > 0$ and $[x]^+ = \max\{x, 0\}$. The payoff of user $i \in \mathcal{N}$ is given by the product of the service quality and its own usage level,

$$u_i(a_0, a) = Q(a_0, a) a_i. \tag{6}$$

The payoff of the manager is given by the average payoff of the users,

$$u_0(a_0, a) = \frac{1}{N} \sum_{i=1}^N u_i(a_0, a).$$

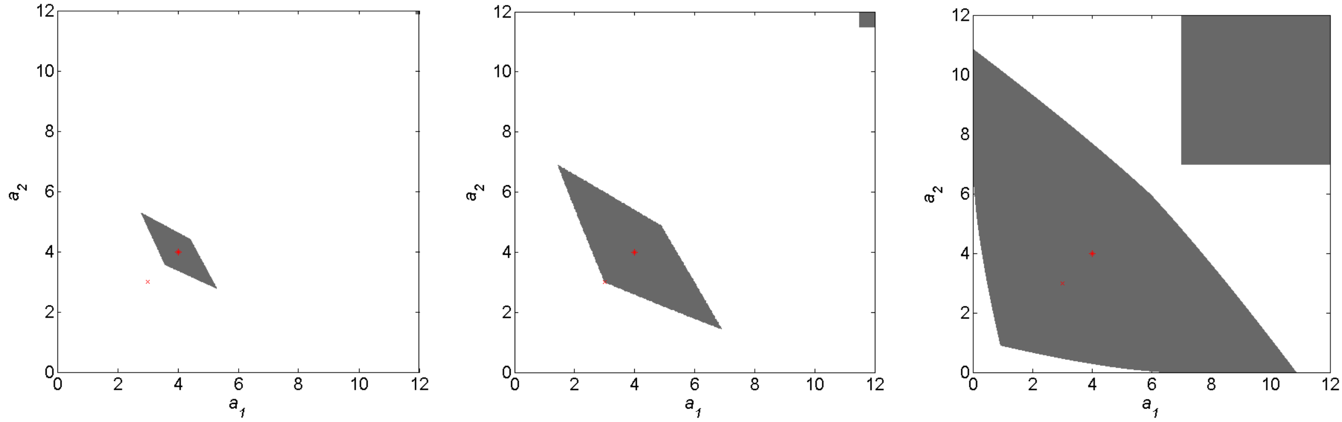
$u_i(a_0, a)$ is weakly decreasing in a_0 for all a, and thus we can consider an extreme intervention rule that takes the value $\overline{a}_0$ whenever a unilateral deviation occurs.

In this example, we have $\overline{v}_0 = q^2/4Nb$, which is achieved when $a_0 = 0$ and $\sum_{i=1}^N a_i = q/2b$. The symmetric action profile that attains $\overline{v}_0$ is thus $(a_l, \ldots, a_l)$, where $a_l \triangleq q/2Nb$. On the other hand, the best performance at the non-cooperative equilibrium without intervention (i.e., when a_0 is held fixed at 0) is given by $\tilde{v}_0 = q^2/(N+1)^2 b$, which is attained at $(a_h, \ldots, a_h)$, where $a_h \triangleq q/(N+1)b$. Note that $a_h > a_l$. Hence, the goal of the manager is to limit the usage levels of the users by using intervention as a threat. In the following proposition, we investigate the best performance with intervention, v_0^*, as we vary $\overline{a}_0$.

Proposition 8: (i) $v_0^* = \tilde{v}_0$ if and only if $\overline{a}_0 = 0$.
(ii) $v_0^* = \overline{v}_0$ if and only if $\overline{a}_0 \geq \overline{a}_0^{min} \triangleq (\sqrt{N} - 1)^2 q/2Nb$.
(iii) v_0^* is strictly increasing with $\overline{a}_0$ on $[0, \overline{a}_0^{min}]$.

Proof: See Appendix B. ■

Since u_i is weakly decreasing in a_0, the set $\mathcal{E}^*$ is weakly expanding as the intervention capability $\overline{a}_0$ is larger. This implies that the performance with intervention v_0^* is weakly increasing with $\overline{a}_0$. Proposition 8 shows that the performance with intervention improves as $\overline{a}_0$ increases, eventually reaching the best feasible performance when $\overline{a}_0 \geq \overline{a}_0^{min}$. Thus, $\overline{a}_0^{min}$ can be interpreted as the minimum intervention capability for an intervention scheme to achieve the best feasible performance. We can show that $\overline{a}_0^{min}$ is increasing and concave in N. Fig. 5 plots the set $\mathcal{E}^* = \mathcal{E}(\mathcal{F})$ as dark regions for the different values of $\overline{a}_0$ with parameters $N = 2$, $q = 12$, $b = 1$, and $\overline{a}_1 = \overline{a}_2 = 12$. We can see that $\mathcal{E}^*$ expands



(a) $\overline{a}_0 = 0.1$ (b) $\overline{a}_0 = 0.51$ (c) $\overline{a}_0 = 5$

Fig. 5. Plot of $\mathcal{E}^*$ as dark regions for the different values of $\overline{a}_0$ in the example in Section IV-B.

as $\overline{a}_0$ increases. When $\overline{a}_0 = 0$, $\mathcal{E}^*$ has only two elements, $(a_h, a_h) = (4, 4)$ and $(12, 12)$. When $\overline{a}_0 = 0.1$, there are more action profiles in $\mathcal{E}^*$. However, the symmetric social optimum $(a_l, a_l) = (3, 3)$ does not belong to $\mathcal{E}^*$, and Proposition 5 implies that the action profile (a_1, a_2) that minimizes $a_1 + a_2$ among those in $\mathcal{E}^*$ constitutes an intervention equilibrium. When $\overline{a}_0 \geq (\sqrt{2} - 1)^2 q/4b \approx 0.51$, the action profiles in $\mathcal{E}^*$ that satisfy $a_1 + a_2 = 2a_l = 6$ constitute an intervention equilibrium, as all of them yield the best feasible performance $\overline{v}_0$. When $\overline{a}_0 \geq q/b = 12$, the punishment from $\overline{a}_0$ is strong enough to make any action profile sustainable, i.e., $\mathcal{E}^* = A$.

Applying Proposition 7, we can construct an affine intervention rule that sustains an action profile a^* such that $a_i^* \in (0, \overline{a}_i)$ for all $i \in \mathcal{N}$ and $\sum_{i=1}^N a_i^* < q/b$, provided that the maximum intervention level $\overline{a}_0$ is sufficiently large. With the payoff functions in (6), the expression of c_i^* in (5) is given by

$$c_i^*(a^*) = \frac{q}{ba_i^*} - \frac{\sum_{j \neq i} a_j^*}{a_i^*} - 2,$$

for all $i \in \mathcal{N}$. For example, the affine intervention rule with target action profile $(a_l, \ldots, a_l)$ and the corresponding intervention rate profile $c^*(a_l, \ldots, a_l)$ is expressed as

$$f(a) = \left[(N-1) \left(\sum_{i=1}^N a_i - \frac{q}{2b} \right) \right]_0^{\overline{a}_0}. \tag{7}$$

Fig. 6 considers $N = 2$ and plots the payoff of user i against its action a_i, provided that the manager chooses the intervention rule in (7) and the other user chooses a_l. It also assumes that $\overline{a}_0$ is sufficiently large. Without intervention, the best response of user i to a_l is $3q/8b$, which shows the instability of the symmetric social optimum (a_l, a_l). However, when the intervention rule (7) is used, the intervention device begins to intervene as user i increases its usage level from a_l. An increase in payoff due to the increased usage level is more than offset by a decrease in payoff due to the quality degradation from intervention. As a result, users do not gain by a unilateral deviation from (a_l, a_l) under the intervention rule (7).

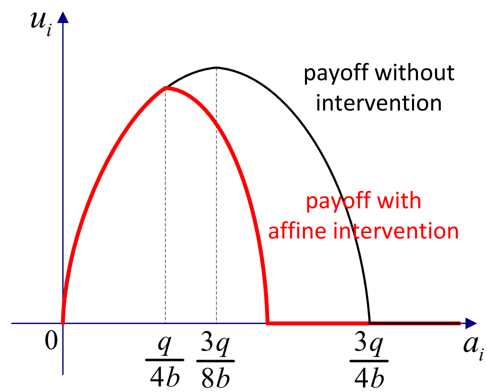


Fig. 6. Plot of u_i against a_i when the manager chooses the affine intervention rule (7) and the other user chooses a_l.

V. Comparison With Existing Approaches

The literature has studied various methods to improve non-cooperative outcomes. One such method is to use contractual agreements. Contract theory is a field of economics that studies how economic actors form contractual agreements, covering the topics of incentives, information, and institutions [16]. Since intervention schemes aim to motivate users to take appropriate actions, our work shares a theme as well as a formal framework with contract theory. However, most works in contract theory deal with the principal-agent problem using monetary payment as the incentive device (see, for example, [17]). In contrast, our work focuses on the problem of regulating selfish behavior in resource sharing by using intervention within the system as the incentive device.

In game theory, correlated equilibrium is a solution concept that extends Nash equilibrium and thus has the potential to improve Nash equilibrium. A correlated equilibrium can be implemented by having a mediator who determines an action profile following a correlated distribution and makes a confidential recommendation to each player [18]. In an intervention game, the manager recommends a pure or mixed action profile to users but does not use a correlated distribution to determine the target action profile. Another difference is that an intervention scheme uses an external punishment device to prevent deviation, which is not present in the concept of correlated equilibrium. We also note that, for the prisoner's dilemma game where there is a dominant strategy for each player, the set of correlated equilibria coincides with that of Nash equilibria. This suggests that correlated equilibrium is more useful for inducing coordination (see, for example, [19], which considers a multiple access network) than for achieving cooperation in a prisoner's dilemma scenario, as considered in this paper.

Another method used in game theory to expand the set of Nash equilibria is repeated games. In a repeated game, players monitor their behavior and choose their actions based on past observations (see, for example, [5] and [20] for works that apply the idea of repeated games to wireless communications). Implementing an incentive scheme based on a repeated game strategy requires long-term relationship among interacting users, which may not exist especially in mobile, cognitive, and vehicular networks. Moreover, a repeated game strategy should be designed in accord with the self-interest of players in order to ensure that they execute monitoring and punishment or reward in a planned way. On the contrary, an intervention scheme uses an external device for monitoring and executing punishment. Hence, it can provide incentives for a dynamically changing population, and the manager can prescribe any feasible intervention rule according to his objective.

In the communications literature, Stackelberg games have been used to improve Nash equilibrium (see, for example, [6] and [21]). Stackelberg games divide users into two groups, a leader and followers, and the leader takes an action before the followers do. In intervention games, the manager is the leader while users are followers, and the manager chooses an intervention rule, which is a contingent plan, instead of an action. Thus, intervention games are more suitable than Stackelberg games when the leader is not a resource user but a manager who regulates resource sharing by users.

Pricing schemes or taxation can also be used to induce individuals to take socially desirable actions. Intervention affects the payoffs of users by directly influencing their resource usage, whereas pricing does so by using an outside instrument, money. Thus, intervention schemes can be implemented more robustly in that users cannot avoid intervention as long as they use resources. In order to achieve a desired outcome through an incentive scheme, the manager needs to know the impact of the incentive device on the payoffs of users. Since intervention affects the payoffs of users through physical quantities associated with resource usage (e.g., throughput, delay), which are easily measurable, this information is easier to obtain when the manager uses an intervention scheme than a pricing scheme, as discussed at the end of Section III-B.

Lastly, we discuss the difference between intervention and mechanism design in the sense of [22, Ch. 23]. In a mechanism design problem, the designer aims to obtain the private information of agents while he can control the social choice (e.g., a resource allocation). On the contrary, in an intervention game, the manager aims to motivate users to take appropriate actions while he has complete information about users (i.e., no private information).

VI. Conclusion and Future Research

In this paper, we have developed a game-theoretic framework for the design and analysis of intervention schemes, which are aimed to drive self-interested users towards a system objective. Our results suggest that the manager can construct an effective intervention scheme when he has an intervention device with an accurate monitoring technology and a strong intervention capability. We have illustrated our framework and results with simple resource sharing scenarios in wireless communications. However, the application of intervention schemes is not limited to the problems considered in this paper; our framework can be applied to a much broader set of problems in communications, including power control and flow control, as well as to various types of networks such as cognitive radio, vehicular networks, peer-to-peer networks, and crowdsourcing websites. Exploring the role of intervention in various specific scenarios is left for future research. Another direction of future research is to combine intervention with other game-theoretic concepts. First, we can introduce intervention in repeated games, where users and the intervention device choose their actions depending on their past observations. We can also allow the intervention manager to use a correlated distribution, as in correlated equilibrium, when he determines the target action profile. Intervention can then be exerted when a user deviates from the recommended action. We can use the idea of mechanism design to deal with a scenario where the intervention manager has incomplete information about users. In such a scenario, the manager first obtains reports from users and then chooses an intervention rule depending on the reports. Finally, intervention can be used in the context of bargaining games, where the set of feasible payoffs in a bargaining game is obtained from sustainable action profiles.

Appendix A
Proof of Proposition 3

Proof: First, note that $w_0(0) > \lim_{\alpha \to 0^+} w_0(\alpha)$. Suppose that $pa_L - qa_H < 0$ and $qa_L - ra_H < 0$. Then $w_0(\alpha) = 0$ for all $\alpha \in (0, 1]$, and thus $v_0^* = w_0(0) = \tilde{v}_0$. This covers condition (a) in Proposition 3. Now suppose that at least one of the two inequalities $pa_L - qa_H \geq 0$ and $qa_L - ra_H \geq 0$ holds. We consider three cases.

Case 1: $pa_L - qa_H = qa_L - ra_H$.

In this case, $\alpha(pa_L - qa_H) + (1 - \alpha)(qa_L - ra_H) \geq 0$ is satisfied for all $\alpha \in (0, 1]$, and $w_0(\alpha)$ is increasing on $(0, 1]$. Thus, we obtain $v_0^* = \max\{w_0(0), w_0(1)\}$.

Case 2: $pa_L - qa_H > qa_L - ra_H$.

$\alpha(pa_L - qa_H) + (1 - \alpha)(qa_L - ra_H) \geq 0$ if and only if

$$\alpha \geq \frac{-(qa_L - ra_H)}{(pa_L - qa_H) - (qa_L - ra_H)}, \tag{8}$$

where the right-hand side of (8) is smaller than 1. Also, $pa_L - qa_H > qa_L - ra_H$ implies $p - q > q - r$. We can show that the sign of the first derivative of w_0 at any $\alpha \in (0, 1)$ is equal to that of $(p - q)(1 - r) - (q - r)(1 - q)$, which is positive. Hence, we have $v_0^* = \max\{w_0(0), w_0(1)\}$. Combining Cases 1 and 2 covers conditions (c) and (d).

Case 3: $pa_L - qa_H < qa_L - ra_H$.

$\alpha(pa_L - qa_H) + (1-\alpha)(qa_L - ra_H) \geq 0$ if and only if $\alpha \leq \overline{\alpha}$. Also, the sign of the first derivative of w_0 is equal to that of $(p-q)(1-r)-(q-r)(1-q)$.

Case 3-1: $0 \leq pa_L - qa_H < qa_L - ra_H$.

We have $\overline{\alpha} \geq 1$. Thus, w_0 is increasing on $(0,1]$ if $(p-q)(1-r)-(q-r)(1-q) > 0$ and non-increasing if $(p-q)(1-r)-(q-r)(1-q) \leq 0$.

Case 3-2: $pa_L - qa_H < 0 \leq qa_L - ra_H$.

We have $\overline{\alpha} < 1$. Thus, w_0 is increasing on $(0,\overline{\alpha}]$ if $(p-q)(1-r)-(q-r)(1-q) > 0$ and non-increasing if $(p-q)(1-r)-(q-r)(1-q) \leq 0$.

These results cover conditions (b), (e), and (f). ■

Appendix B
Proof of Proposition 8

Proof: (Sketch) Note that $u_0(0,a)$ depends on a only through $\sum_{i=1}^N a_i$. $u_0(0,a)$ is increasing in $\sum_{i=1}^N a_i$ for $0 \leq \sum_{i=1}^N a_i \leq q/2b$, reaches the maximum at $\sum_{i=1}^N a_i = q/2b$, is decreasing in $\sum_{i=1}^N a_i$ for $q/2b \leq \sum_{i=1}^N a_i \leq q/b$, and remains at zero for $\sum_{i=1}^N a_i \geq q/b$.

(i) If $\overline{a}_0 = 0$, then $v_0^* = \tilde{v}_0$ by definition. To show the converse, suppose that $\overline{a}_0 > 0$. Since the payoff function is continuous, we can show that $(a_h-\epsilon,\ldots,a_h-\epsilon)$ is sustainable for sufficiently small $\epsilon > 0$, which yields $v_0^* > \tilde{v}_0$.

(ii) We have $v_0^* = \overline{v}_0$ if and only if there exists a sustainable action profile a such that $\sum_{i=1}^N a_i = q/2b$. Given $\sum_{i=1}^N a_i = q/2b$, the incentive for user i to deviate is stronger as a_i is smaller. Hence, it suffices to check whether the symmetric action profile $(a_l,\ldots,a_l)$ is sustainable. By Lemma 3, $(a_l,\ldots,a_l)$ is sustainable if and only if

$$\max_{a_i \in [0,\overline{a}_i]} [q - b(\overline{a}_0 + (N-1)a_l + a_i)]^+ a_i \leq q^2/4Nb,$$

which is equivalent to $\overline{a}_0 \geq (\sqrt{N}-1)^2 q/2Nb$.

(iii) Choose $\overline{a}_0, \overline{a}_0' \in [0, \overline{a}_0^{min}]$ with $\overline{a}_0 < \overline{a}_0'$. Let v_0^* and $(v_0^*)'$ be the corresponding performances with intervention. Since $0 \leq \overline{a}_0 < \overline{a}_0^{min}$, there exists an action profile a that attains v_0^* with intervention capability $\overline{a}_0$ and satisfies $q/2b < \sum_{i=1}^N a_i \leq Nq/(N+1)b$. We can show that $(a_1-\epsilon,\ldots,a_N-\epsilon)$ can be sustained with $\overline{a}_0'$ for sufficiently small $\epsilon > 0$, which implies $(v_0^*)' > v_0^*$. ■

References

[1] J. Park and M. van der Schaar, "Incentive provision using intervention," in *Proc. INFOCOM*, 2011, pp. 571–575.

[2] G. Hardin, "The tragedy of the commons," *Science*, vol. 162, no. 3859, pp. 1243–1248, Dec. 1968.

[3] M. Felegyhazi and J.-P. Hubaux, "Game theory in wireless networks: a tutorial," *EPFL Technical Report*, LCA-REPORT-2006-002, Feb. 2006.

[4] A. Laufer and A. Leshem, "Distributed coordination of spectrum and the prisoner's dilemma," in *Proc. DySPAN*, 2005.

[5] M. Čagalj, S. Ganeriwal, I. Aad, and J.-P. Hubaux, "On selfish behavior in CSMA/CA networks," in *Proc. INFOCOM*, Miami, FL, Mar. 2005.

[6] R. T. Ma, V. Misra, and D. Rubenstein, "An analysis of generalized slotted-Aloha protocols," *IEEE/ACM Trans. Netw.*, vol. 17, no. 3, pp. 936–949, Jun. 2009.

[7] J. Park and M. van der Schaar, "Stackelberg contention games in multiuser networks," *EURASIP J. Advances Signal Process.*, vol. 2009, Article ID 305978, 15 pages, 2009.

[8] J. Park and M. van der Schaar, "Designing incentive schemes based on intervention: The case of imperfect monitoring," in *Proc. GameNets*, 2011.

[9] R. Garg, A. Kamra, and V. Khurana, "A game-theoretic approach towards congestion control in communication networks," *Comput. Commun. Review*, vol. 32, no. 3, pp. 47–61, Jul. 2002.

[10] Y. Gai, H. Liu, and B. Krishnamachari, "A packet dropping-based incentive mechanism for M/M/1 queues with selfish users," in *Proc. IEEE INFOCOM*, 2011.

[11] D. Fudenberg and J. Tirole, *Game Theory*. Cambridge, MA: MIT Press, 1991.

[12] J. F. Nash, "Equilibrium points in n-person games," *Proc. Nat. Academy Sci.*, vol. 36, no. 1, pp. 48–49, Jan. 1950.

[13] N. L. Stokey and R. E. Lucas (with E. C. Prescott), *Recursive Methods in Economic Dynamics*. Cambridge, MA: Harvard Univ. Press, 1989.

[14] M. Kandori and I. Obara, "Efficiency in repeated games revisited: the role of private strategies," *Econometrica*, vol. 74, no. 2, pp. 499–519, Mar. 2006.

[15] D. Abreu, "On the theory of infinitely repeated games with discounting," *Econometrica*, vol. 56, no. 2, pp. 383–396 , Mar. 1988.

[16] P. Bolton and M. Dewatripont, *Contract Theory*. Cambridge, MA: MIT Press, 2005.

[17] B. Holmstrom, "Moral hazard in teams," *Bell J. Econ.*, vol. 13, no. 2, pp. 324–340, Autumn 1982.

[18] R. Myerson, *Game Theory: Analysis of Conflict*. Cambridge, MA: Harvard Univ. Press, 1991.

[19] E. Altman, N. Bonneau, and M. Debbah, "Correlated equilibrium in access control for wireless communications," in *Proc. Networking*, 2006, pp.173–183.

[20] S. Buchegger and J.-Y. Le Boudec, "Performance analysis of the CONFIDANT protocol (Cooperation Of Nodes: Fairness In Dynamic Ad-hoc NeTworks)," in *Proc. MobiHoc*, 2002, pp.226–236.

[21] Y. A. Korilis, A. A. Lazar, and A. Orda, "Achieving network optima using Stackelberg routing strategies," *IEEE/ACM Trans. Netw.*, vol. 5, no. 1, pp. 161–173, Feb. 1997.

[22] A. Mas-Colell, M. D. Whinston, and J. R. Green, *Microeconomic Theory*, Oxford, U.K.: Oxford Univ. Press, 1995.

Jaeok Park received the B.A. degree in economics from Yonsei University, Seoul, Korea, in 2003, and the M.A. and Ph.D. degrees in economics from the University of California, Los Angeles, in 2005 and 2009, respectively.

He is currently an Assistant Professor in the School of Economics at Yonsei University, Seoul, Korea. From 2009 to 2011, he was a Postdoctoral Scholar in the Electrical Engineering Department at the University of California, Los Angeles. From 2006 to 2008, he served in the Republic of Korea Army. His research interests include game theory, mechanism design, network economics, and wireless communication.

Mihaela van der Schaar (F'10) is Chancellor's Professor of Electrical Engineering at the University of California, Los Angeles. She is a Distinguished Lecturer of the Communications Society for 2011–2012, the Editor in Chief of IEEE Transactions on Multimedia and a member of the Editorial Board of IEEE Journal on Selected Topics in Signal Processing. She holds 33 granted US patents and received several best paper awards. More information can be found at http://medianetlab.ee.ucla.edu/.