

Token System Design for Autonomic Wireless Relay Networks

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Abstract—This paper proposes a novel framework for incentivizing self-interested transceivers operating in autonomic wireless networks to provide relaying services to other transceivers in exchange for tokens. Tokens represent a simple internal currency which can be used by the transceivers in a network to exchange services. Our emphasis in this paper is on developing optimal designs for the token system, which maximize the system efficiency, i.e. the probability that the relay transmission will be executed by transceivers whenever they are requested to provide such services. Particularly, we prove that the efficiency of the relay network heavily depends on issuing the proper amount of tokens rather than an arbitrary amount. First, we study the transceivers’ optimal strategies (i.e. the strategies that maximize the transceivers’ own utilities) using the formalism of repeated games. We prove that these strategies exhibit a simple threshold structure. We also prove that the threshold is unique given transmission costs. Second, we determine the optimal token amount which needs to be introduced in the relay system to maximize the overall relay network efficiency. This amount needs to be neither too small (since a too small amount leads to a small relaying service request probability) nor too large (since a too large amount leads to a small relaying service provision probability) and depends on the threshold strategy that the self-interested transceivers adopt. We subsequently develop an efficient algorithm which is able to determine, depending on the network characteristics, the threshold to be implemented by the optimal strategies and the optimal token amount. Finally, simulation results show the effectiveness of our token system design in providing incentives for cooperation among self-interested relays in autonomic wireless relay networks.

Index Terms—autonomic communication, wireless relay networks, cooperative communication, incentives, tokens, repeated games.

I. INTRODUCTION

In many wireless communication scenarios, the transmission between two distant transceivers may have to be accomplished with the help of an intermediate node (i.e. relay) due to limited transmit power, deep fading exhibited by the direct transmission channel or other transmission constraints [1]. The concept of relaying has been adopted in many wireless infrastructures to improve the transmission quality, such as multi-hop ad hoc networks, multi-hop cellular networks, wireless cooperative networks as well as next generation telecommunication standards (e.g. IEEE 802.16j). To maximize the overall network utility, most existing works that address the resource allocation problem in wireless relay networks assume that (1) there is a centralized agency (e.g. base stations) that performs the resource allocation, (2) the network scale is small so the

resource allocation problem can be relatively easily solved and (3) all the participating devices will obediently follow the prescribed allocation policy. However, these assumptions do not hold for *autonomic* wireless relay networks where (1) the network infrastructure is fully distributed and thus, a central agency cannot intervene or manage the individual transceivers’ behaviors at run-time, (2) the network can consist of numerous anonymous transceivers, and (3) the participating devices (or their owners) are self-interested and may refuse to cooperatively relay traffic for other devices while incurring a cost (e.g. relay transmission power).

Various incentive schemes have been recently proposed in the literature to stimulate relay cooperation in wireless relay networks. One way to foster cooperation is by using monetary pricing schemes [11][12]. These methods often focus on static networks where the interacting transceivers are fixed. A key disadvantage of using monetary pricing is the absence of the associated reliable financial accounting and the impracticality of implementing centralized accounting to pay for decentralized services (i.e. relay cooperation among distributed transceivers). Moreover, these methods are only suitable for small-scale networks but they are difficult to implement in a large-scale autonomic wireless relay system.

Another strand of literature proposes using reputation-based methods for rewarding or punishing a transceiver based on its behavior in the system. In [2], the “watchdog” mechanism is proposed to identify the misbehaving nodes and the “pathrater” mechanism is used to deflect the traffics around them in mobile ad hoc networks. To enforce the cooperation by punishing misbehaving nodes, in [3] and [4] reputation-based mechanisms are proposed to enforce node cooperation. A mathematical analysis of the interaction emerging among relays using game theory is provided in [5][6][7]. Various versions of the well-known Tit-For-Tat strategy are proposed in [8][9]. Moreover, using the repeated game framework, reputation schemes were also proposed for providing incentives in relay networks where nodes periodically update their partners due to mobility or changes of environment [10]. However, a significant limitation of reputation-based schemes when deployed in autonomic wireless networks is that they rely on a central agency that is able to collect, process, and deliver information about individuals’ behavior. This is because in order for these reputation schemes to work, transceivers need to know the reputations of their partners. Establishing such an infrastructure for managing the reputations of transceivers may be prohibitively costly and also, it would require that the transceivers can be identified (they can no longer remain anonymous, as required in most deployments of autonomous wireless networks). Summarizing, existing works either require a central agency to intervene and

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manage the individual transceivers' behavior in real-time or do not scale for easy deployment in large-scale systems.

In this paper, we build a token system [13] to provide transceivers in autonomic wireless relay networks with incentives to provide relaying service due to its implementation simplicity and possibility to operate the network in an autonomous and distributed way. We will show that it is possible to design effective incentive schemes for distributed and large-scale wireless relay networks using token systems.

Electronic tokens are not new and were proposed to provide incentives in wireless relay networks [14] as well as other peer-to-peer systems [17][18][19][20]. Basically, electronic tokens work as virtual currency: transceivers pay tokens to relays in exchange for the provided relaying services. In [14], Nuglets are introduced as the virtual currency in wireless ad hoc networks. In networks operating based on Nuglets, one Nuglet is transferred from the sender to the relay transceiver for forwarding a message. Following this work, a tamper-proof solution is developed in [15] focusing on the security issues involved when implementing such systems. However, existing works focus on the token implementation or security aspects and rely on simulation methods to quantify the performance of their proposed design. They lack a rigorous model and performance analysis of the emerging relay interactions. Most importantly, users' strategic behavior in token systems is not systematically analyzed, thereby leading to significant degradation in the token system performance. Analytical attempts to understand the users' strategic behavior and its effect on the system efficiency are made in [21] but this work is focused on a very different network deployment scenario. The major differences are: (1) [21] studies a deployment scenario where only one agent makes a service request in each period; (2) [21] assumes that the agent strategy is a threshold strategy and shows that threshold strategies can be possible Nash equilibria but it does not exclude the possibility that non-threshold strategy can also be a Nash equilibrium; (3) [21] does not establish the optimal token supply.

In this paper, we provide a rigorous analysis of our proposed token system and prove its optimality. Importantly, we prove that the efficiency of the relay network heavily depends on issuing the proper amount of tokens rather than an arbitrary amount. To determine this optimal amount, the system designer needs to understand the self-interested behavior of the strategic transceivers. We formulate the transceivers' strategic behavior using a novel repeated game formalism and prove that transceiver strategies exhibit a threshold structure. Then we show that there exists an optimal token amount which needs to be introduced in the system for it to operate optimally. This needs to be neither too small (since a too small amount leads to a small relaying service request probability) nor too large (since a too large amount leads to a small relaying service provision probability) and depends on the threshold strategy that self-interested transceivers adopt. Subsequently, an efficient algorithm is developed to compute the optimal threshold and determine the optimal token amount depending on the network characteristics. Table I succinctly compares the proposed token systems with the existing literature.

The rest of this paper is organized as follows. Section II

<i>Characteristics</i>	<i>NUM</i>	<i>Pricing</i>	<i>Reputation</i>	<i>(Existing) Token</i>	<i>Proposed</i>
<i>Distributed network</i>	No	No	No	Yes	Yes
<i>Large scale network</i>	No	No	No	Yes	Yes
<i>Self-interested agents</i>	No	Yes	Yes	Yes/NO	Yes
<i>Anonymous agents</i>	No	No	No	Yes	Yes
<i>Rigorous design</i>	Yes	Yes	Yes/No	No	Yes

TABLE I
COMPARISON WITH EXISTING WORKS

introduces the system model and describes the relay transmission process. Section III studies the optimal strategies for the transceivers and shows that only threshold strategies are optimal. Moreover, for each relaying cost, there exists a unique associated optimal threshold. Section IV then determines the optimal token supply that maximizes the system efficiency. An efficient bisection algorithm is provided to find the threshold that transceivers adopt. Section V provides simulation results. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

A. Setup

We consider a dynamic wireless network with N wireless mobile transceivers. We consider N to be large since there are usually many transceivers in the network. In each time period, a fraction of the transceivers needs to receive data from their corresponding sources (e.g. the base station in a cellular network or another nearby mobile transceiver in an ad hoc network). However, not all of these transmissions can be fulfilled simply through direct transmissions since the wireless channel between the sources and the destinations may be degraded, e.g. exhibit deep fading or shadowing. In such cases, relay transmissions via intermediate transceivers which forward signals to the destination are required to improve the network performance [1].

At this point, several points regarding the operating of the considered autonomic wireless relay network are worth noting:

- 1) Transceivers are mobile and hence, they move to various locations at different times. Hence, they have different neighboring transceivers and experience different channel conditions at different times.
- 2) The transceivers that need relay transmissions are different in different time periods. This depends both on the transmission demand arrival process of the transceivers and the realizations of the channel conditions for the specific transmissions.

We capture the demand for relay transmissions in the network by λ , which is the probability that a transceiver needs to receive data from its source using relay transmissions in each period. Hence, λ is the relay transmission demand rate and depends on the overall network condition. Note that for individual transceivers, the relay transmission demand probabilities may be different. In the simulation section, we will show that using the mean network relay demand probabilities λ achieves close-to-optimal performance even when individual

transceivers' relay transmission demand rates are heterogeneous. We assume that the transceivers are anonymous and self-interested, meaning that they aim to maximize only their own utilities and do not care about the overall performance of the network. Because forwarding signals incurs costs (e.g. transmission power) to the transceivers who act as relays, self-interested transceivers do not want to help other transceivers by forwarding their traffic without having proper incentives. Hence, our focus is on designing such incentive mechanisms. Denote the action space of the transceivers which are requested for relay service by $\mathcal{A} = \{0, 1\}$; the action $a = 0$ means "not relay" and $a = 1$ means "relay".

Suppose at time period t , a transceiver j acts as the relay and a transceiver i is the receiver. Note that we allow multiple simultaneous transmissions and hence, there may be multiple relay-receiver pairs in the same time slot in the relay network. Nevertheless, we focus on one such pair for illustration. If transceiver j does relay for transceiver i , transceiver i (as the receiver) enjoys a benefit $b(r_d)$ which depends on the receiving data rate r_d . Transceiver j (as the relay) incurs a cost $c(r_d, G_{sr}, G_{rd})$ which depends on the data rate r_d as well as the channel conditions to conduct the relay transmission at this transmission rate, i.e. the channel gain between the source and the relay G_{sr} and the channel gain between the relay and the destination G_{rd} . Though the cost incurred when relaying traffic is affected by many considerations, in this paper, we specifically consider the relay transmission power as the cost to the relay transceiver to achieve a target transmission rate r^{target} . Suppose the source transmission power is fixed at P_s and denote the relay transmission power as P_r . Standard relay channel analysis for Amplify-and-Forward (AF)¹ yields a received Signal-to-Interference-and-Noise Ratio (SINR) as

$$\Gamma_{srd} = \frac{\Gamma_{sr}\Gamma_{rd}}{\Gamma_{sr} + \Gamma_{rd} + 1} \quad (1)$$

where Γ_{sr} and Γ_{rd} are the SINRs on the source-relay channel and the relay-destination channel. To achieve a certain target rate r^{target} for the receiver transceivers, it is equivalent that the SINR is larger than a corresponding target value Γ^{target} . Hence, the minimum required relay transmission power is

$$P_r = \arg \min \{P_r : \Gamma_{srd} \geq \Gamma^{target}\} \quad (2)$$

Therefore, the relaying cost $c(r^{target}, G_{sr}, G_{rd})$ given a target rate is the solution to (2). Depending on which application is running on the receiver, the target transmission rates of the transceivers may vary over time. Let $b = \mathbb{E}b(r^{target})$ be the expected benefit over all possible target rates. We normalize the costs to this expected benefit (i.e. by dividing the cost by b).

Note that since the channel condition realizations for different transmission pairs are different, the associated costs for the relay transceivers also vary. However, since the cost is always positive (i.e., $c(r^{target}, G_{sr}, G_{rd}) > 0$), the dominant strategy for the relay transceiver is always to not forward traffic in this simple gift-giving game (see Figure 1).

¹Our analysis is also applicable to other relay schemes other than AF.

	Relay Transceiver	
Receiving Transceiver	$b, -c$	$(0, 0)$
	<i>Relay</i>	<i>Not Relay</i>

Fig. 1. The relay transmission game. (first utility - receiving transceiver; second utility - relay transceiver)

B. Token System

If the transceivers would only be involved in a single relay transmission as the relay, they will be reluctant to help forward the traffic since this incurs them a cost and provides them with no reward. However, because transceivers are active in the network for a long time, proper incentives can be provided to make them take into account when making decisions the benefits of relaying. We assume that the transceivers discount the future utility at a constant rate $\beta \in (0, 1]$ ². One way to introduce such incentives to relay traffic for other transceivers is through the use of tokens, which are exchanged among transceivers in order to "buy" and "sell" relaying services. In each relay transmission, the receiving transceiver pays one token to the relay transceiver in exchange for forwarding the traffic.

The overlay token system enables simple deployment of the relay network with self-interested transceivers. (1) One token provides one unit relay transmission opportunity and has no intrinsic value outside of the relay network. This avoids many financial problems (such as fraud) that are associated with monetary incentive schemes. (2) No personal information of the others is required when a transceiver makes a decision. Hence, the system can be fully anonymous and more secure. (3) Several techniques that enable secure electronic token transactions in a distributed way have been proposed [30][14]. (Essentially no central entity is needed for the transactions.) Our work assumes using such technologies for implementing the proposed token exchange protocol.

C. Timing of the relay request and transmission

The conventional relay transmission process often involves two stages: relay selection and transmission. However, besides the conventional two stages for the relay transmission in each period with obedient transceivers, there is one more decision stage in the presence of self-interested transceivers because they have to decide whether relaying the traffic is in their best interests. In the relay selection stage, the receiving transceiver selects a neighbor transceiver as the candidate relay and sends a relay request message (REQ). When the candidate relay receives the REQ, it makes the decision on whether or not to provide the relaying service. If it accepts the request, it sends back an acceptance acknowledgement (ACK). Then the relay transmission follows in the transmission stage. If the candidate relay declines the request, it sends back a decline acknowledge (NACK). In such a scenario, either no transmission or simply direction transmission follows in the transmission stage. To deploy such a system, two essential

²One interpretation of the discount factor β is the probability that the transceivers stay in the network. For example, if $\beta = 0.9$, the transceivers stay in the network with probability 0.9 in the next period.

issues for its implementation need to be emphasized. (Note though that these implementation aspects do not affect the proposed token system design.)

First, relay selection has been shown to be critical for the relay network performance and much work has focused on this issue [22][23][24][25]. However, since the main focus of this paper is not on how to select the optimal relay but rather on how to incentivize the relay transceiver to provide the relaying service once it is selected, we only briefly describe several relay selection solutions which can be adopted in conjunction with the proposed token system. The simplest relay selection scheme is the random selection, i.e. the receiving transceiver randomly selects a neighbor transceiver as the candidate relay. More sophisticated relay selection schemes, in which the receiving transceiver gathers the (partial) channel information of its neighbor transceivers (e.g. through beacons on the control channel) and estimates the required relay transmission power (e.g. based on the received signal strength of the beacon), can also be employed. Subsequently, the receiver chooses the transceiver requiring the least relay transmission power. If efficient channel assignment schemes are available (e.g. see [27][28]), almost all transmissions in the network can take place on orthogonal channels within interference regions, then the relay selection criterion is simply to choose the wireless transceiver with the best channel condition and thus requiring the least power for relaying. Otherwise, co-channel interference may influence the relay transmission performance. In such scenarios, the receiving transceiver may use approximate relay selection metrics (e.g. ignoring the potential interference when calculating the required power) or some interference-aware metrics (e.g. estimating the potential interference [24][25]) to perform the relay selection. However, it is important to note that the transceivers are not able to observe the number of tokens that other transceivers have due to privacy considerations and hence, it cannot choose the candidate relay according to the token holding.

Second, solutions for implementing token passing securely and efficiently among anonymous transceivers are essential. Our token exchange system will rely on existing solutions for token exchanges proposed for e-commerce e.g. [29][30][31]. A common method to ensure that tokens are securely exchanged involves a Trusted Third Party (TTP), i.e. an escrow service. However, such solutions are centralized and cannot be readily deployed in autonomic relay networks, which are inherently distributed. Alternatively, distributed fair exchange protocols which do not require a centralized TTP are proposed in [29]. Our proposed token system is based on the solution in [29].

We assume that once the candidate relay transceiver accepts a relay request, a secure communication channel is setup between the receiving transceiver and the relay transceiver. This could be easily done using various authentication and encryption methods. Therefore, the token passing is protected from the outside attacks. Moreover, each transceiver participating in the wireless relay networks using tokens is equipped with a tamper-proof secure module (SM). Transceivers cannot have access to the stored data or change the secure module's behavior. However, full access stays possible, but limited to some authorized parties, i.e. the network provider. In this way,

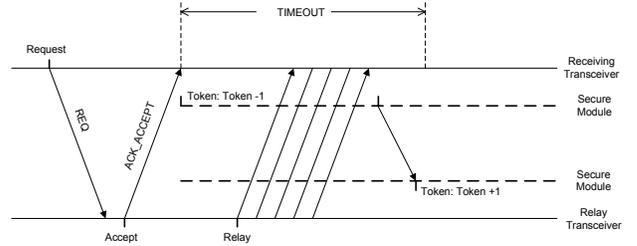


Fig. 2. Secure Token Passing Process.

the SM plays the role of a distributed TTP. Upon receiving the ACK from the relay transceiver, the token passing works as follows: (1) The receiving transceiver's SM decreases its token counter by 1. (2) The relay transceiver relays the packets to the receiving transceiver's SM. (3) If the desired packets are received before timeout, the SM sends an encrypted message to the relay transceivers' SM which then increases its token counter by 1. If the desired packets are not received, then the receiving transceiver's SM increases its token number by 1. Figure 2 illustrates the described secure token passing process.

D. Problem Formulation

To obtain good system performance for the token system (i.e., a high probability that the relay transmission is accepted and executed when requested by transceiver), a careful design by the system designer is required. A key question when designing such a system is what the amount of tokens that are circulated in the system should be. A straightforward intuition is that the token system does not work if there are too few tokens in the network because few transceivers have the tokens to request relay transmissions. In addition, in this paper, we will also show that too many tokens are not helpful either, by studying the strategies that the self-interested transceivers use. Therefore, there must be a proper number of tokens that the designer must deploy in the network.

Denote the transceiver strategy by $\sigma : \mathcal{S} \rightarrow \mathcal{A}$, which is a mapping from the system state space \mathcal{S} to the relay action space \mathcal{A} . Each state $s \in \mathcal{S}$ captures a combination of channel states and token holdings of all possible transmission pairs. Transceivers may use different strategies and hence, we denote σ_i as transceiver i 's relaying strategy. Denote the total amount of tokens circulating in the system by W . The efficiency, which is the expected probability that the relay transmission successfully takes place over the states, is denoted by $\mathbb{E}_{\mathcal{S}}\{E(\sigma_1, \sigma_2, \dots, \sigma_N, s|W)\}$, where $E(\sigma_1, \sigma_2, \dots, \sigma_N, s|W)$ is the relay transmission probability in system state s when transceiver i uses action $\sigma_i(s)$, $\forall i$ in state $s \in \mathcal{S}$. The objective of the system designer is to issue a proper number of tokens in the system such that the relay transmission probability is maximized when all transceivers play optimal strategies to maximize their own utilities. Hence, the designer's problem can be formulated as a hierarchical problem as follows.

The transceiver-level problem deals with transceivers' incentives and tries to solve the optimal transceiver strategy, i.e. the strategy that maximizes transceivers' own utilities. This problem is solved by the self-interested transceivers. Denote $V_i(s|\sigma_i)$ as the long-term utility when transceiver i is in a state $s \in \mathcal{S}$ and uses the strategy σ_i . The transceiver-level

problem is to find the optimal strategy σ_i such that $\forall s \in \mathcal{S}$, $V_i(s|\sigma_i) \geq V_i(s|\sigma'_i), \forall \sigma'_i \neq \sigma_i$. Therefore, the output of this problem is a transceiver strategy σ_i that determines the optimal actions for all states.

The designer-level problem is to maximize the system efficiency by issuing the optimal amount of tokens into the system. This problem is solved by the system designer. However, the designer can solve this problem only after understanding the self-interested behavior of the transceivers and hence, the designer also needs to solve the transceiver-level problem in the first place. Therefore, the designer-level problem is as follows,

$$\begin{aligned} & \underset{W}{\text{maximize}} && \mathbb{E}_{\mathcal{S}}\{E(\sigma_1, \sigma_2, \dots, \sigma_N, s|W)\} \\ & \text{subject to} && \sigma_i \text{ is the optimal transceiver strategy} \\ & && \text{by solving the transceiver-level problem, } \forall i \end{aligned}$$

III. OPTIMAL TRANSCEIVER STRATEGIES

A self-interested transceiver tries to maximize its own utility when making a decision on whether or not to forward the traffic. Suppose the transceiver already has k tokens. If the transceiver decides to forward the traffic, it will gain one more token to make the total number be $k + 1$ in the next time period; otherwise, it remains having k tokens in the next time period. Because forwarding the traffic incurs an instant cost c , the transceiver needs to compare the marginal utility $V(k + 1|\sigma) - V(k|\sigma)$ with this cost to make a utility maximization decision, where $V(k|\sigma)$ is the utility of holding k tokens. For different strategies σ , the induced utilities $V(k|\sigma), \forall k \in \mathbb{N}$ are different. Therefore, a relay strategy is optimal, meaning that the transceiver would like to follow the strategy, if and only if it has the one-shot deviation property [26]:

Definition 1: (Optimal Strategy). A transceiver strategy σ is an optimal strategy if and only if $\forall k \in \mathbb{N}$,

$$\begin{aligned} \beta(V(k+1|\sigma) - V(k|\sigma)) &\geq c, \text{ if } \sigma(k) = 1 \\ \beta(V(k+1|\sigma) - V(k|\sigma)) &< c, \text{ if } \sigma(k) = 0 \end{aligned} \quad (3)$$

Note that the discount factor β is applied since the marginal utility is obtained in the next period. The optimal strategies make the transceiver always maximize its utility for all possible token numbers that it might have by following the strategy. The number of all possible transceiver strategies is large and hence, finding the optimal strategies is difficult. In the following we study whether a strategy σ is optimal and simply write $V(k)$ instead of $V(k|\sigma)$ for brevity.

A. Values and Marginal Values

The value of holding tokens depends on the strategy that the transceiver uses. The utility functions are inter-dependent with each other as follows

$$\begin{aligned} V(0) &= (1 - \lambda\sigma(0))\beta V(0) + \lambda\sigma(0)(-c + \beta V(1)) \\ V(k) &= \underbrace{\lambda(b + \beta V(k-1))}_{\text{Loose one token}} + \underbrace{\lambda\sigma(k)(-c + \beta V(k+1))}_{\text{Obtain one more token}} \\ &\quad + \underbrace{(1 - \lambda(1 + \sigma(k)))\beta V(k)}_{\text{The same token number}}, \forall k \geq 1 \end{aligned} \quad (4)$$

For $k \geq 1$, in each period, with probability λ , the transceiver becomes a receiving transceiver which is in need of relay service. In this case, it spends one token and obtains a benefit b by receiving the service. With probability λ it becomes a relay transceiver. If the strategy is to relay, then it obtains one more token and incurs a relaying cost c . So the expected long-term utility is the second term. With probability $\lambda(1 - \sigma(k))$, the transceiver does not provide service when it is a relay. Moreover, with probability $(1 - 2\lambda)$, it is idle. Therefore, with probability $(1 - \lambda(1 + \sigma(k)))$, the transceiver keeps the same number of tokens. This is the third term. For $k = 0$, the long-term utility can also be similarly analyzed. The only difference is that the transceiver cannot request the relaying service in any case since it has no token.

It is convenient to denote the marginal utility $V(k+1) - V(k)$ of holding k tokens as $M(k)$. In the following, we study the property of the marginal utilities. We denote K_σ as the smallest level k that makes $\sigma(k) = 0$, i.e. ‘‘not relay’’. More precisely, for every transceiver strategy σ there exists $K_\sigma \geq 0$ such that $\sigma(k) = 1, \forall k < K_\sigma$ and $\sigma(K_\sigma) = 0$. However, for $k \geq K_\sigma$, $\sigma(k)$ can be arbitrary. We first study the marginal utilities for $k \leq K_\sigma$ in the following lemma.

Lemma 1: For any transceiver strategy, the marginal utilities satisfy

- 1) If $0 \leq k \leq K_\sigma$, then $M(k) > 0$.
- 2) In the range $0 \leq k \leq K_\sigma$, M is either increasing, decreasing or decreasing the increasing.
- 3) If $M(K_\sigma - 1) \geq c/\beta$, then $M(0) > M(1) > \dots > M(K_\sigma - 1) \geq c/\beta$.

Lemma shows how the marginal utilities look like. For any strategy, the marginal utilities below K_σ are positive, meaning that having more tokens always generates a higher utility. More importantly, if the strategy is an optimal strategy, by the third part of this lemma, the marginal utility diminishes with an increase in the token holding. This implies that transceivers may not want to accumulate more tokens beyond some point if the marginal utility of having one more token falls below the current cost. However, because Lemma only partially studies the marginal utilities (i.e. those below K_σ), we establish this threshold property in the next subsection by studying the general case.

B. Threshold property

We now study which transceiver strategies can be optimal strategies.

Proposition 1: An optimal transceiver strategy σ is a threshold strategy, i.e., there exists K_{th} , such that

$$\begin{aligned} \sigma(k) &= 1, \text{ for } k \leq K_{th} \\ \sigma(k) &= 0, \text{ for } k > K_{th} \end{aligned} \quad (5)$$

The above proposition tells that the optimal strategies can only be threshold strategies. This tremendously simplifies our analysis on transceivers’ rational behaviors by only focusing on the thresholds. Our intuition also suggests that transceivers may like to use threshold strategies due to their simplicity and in fact, many research works make this assumption when they build their models. Different from these works, we start

from arbitrary relay strategies and analytically prove that threshold strategies are indeed the rational optimal choices of the transceivers instead of just assuming this property due to its simplicity without rigorously proving its optimality. Using this novel repeated game formalism, we are also able to determine the thresholds that transceivers may want to use. We discuss how to determine the threshold in Section IV, when we study the optimal token supply.

C. Varying cost

Now we take into account the impact of the instant cost c on the transceiver's decision problem. Because the costs to the relay transceivers are different over time due to changing locations and varying channel conditions, transceivers may not want to use a constant threshold strategy. In the following, we study how the cost for relaying affects the choice of threshold.

Proposition 2: For given λ, β, b and a threshold strategy σ_K with threshold K , there exists c^L, c^H ($0 < c^L < c^H$) such that $\forall c \in (c^L, c^H]$, σ_K is an optimal strategy; otherwise, it is not.

Proposition 2 states that there is a corresponding continuous interval of cost values that make a threshold strategy be optimal. It establishes the conditions that need to be satisfied by the cost for a threshold strategy to be optimal. However, we are more interested in, for a given cost, whether there exists a (or some) threshold strategy to make it optimal and this is not obvious by Proposition 2.

Proposition 3: There exists a maximal value of the cost c_0 such that $\forall 0 < c \leq c_0$, there exists a unique K , such that σ_K is the optimal strategy for c .

Proposition 3 suggests that there is a mapping from the costs to the threshold strategies such that the threshold strategies are optimal. Importantly, the threshold is unique for any cost. Denote the mapping by $\mathcal{K} : (0, c_0] \rightarrow \mathbb{N}$. This is important for understanding the relay transceivers' strategic behavior when they need to decide whether or not to forward traffic at a cost when they already have a certain number of tokens. Hence, the transceiver takes joint considerations of the number k of tokens that it already has and the cost c that incurs by relaying the traffic. The optimal strategy is then a mapping from $\sigma : \mathbb{N} \times (0, c_0] \rightarrow \mathcal{A}$, and

$$\sigma(k, c) = \begin{cases} 1, & \text{if } k < \mathcal{K}(c) \\ 0, & \text{if } k \geq \mathcal{K}(c) \end{cases} \quad (6)$$

IV. OPTIMAL TOKEN SUPPLY

In the previous section, we show that relay transceivers do not cooperate, i.e. forward the traffic all the time, because they have incentives to stop accumulating tokens after accumulating a certain treasury. This suggests that if all transceivers already have many tokens, they stop forwarding traffic when they become relays. On the contrary, it is obvious that if there are too few tokens in the network, relay requests are seldom initiated because few transceivers have tokens to pay when they are receivers. Therefore, it seems there must be an optimal token supply in the network that maximizes the system efficiency, i.e. the probability that a relay transmission successfully takes place when needed.

Because the transceiver population is usually very large, we approximate it by a continuum model (mass 1). Under this continuum model, the token supply is described by the average token number per transceiver $\alpha = W/N$. Let $\eta_K(k)$ be the fraction of relay transceivers who has k tokens and the cost for whom to relay traffic is $c \in \{c : \mathcal{K}(c) = K\}$, then the fraction of relay transceivers who deny forwarding traffic is calculated by

$$\eta_d = \sum_{i=0}^{\infty} w(i) \sum_{k \geq i} \eta_i(k) \quad (7)$$

where $w(k)$ is the fraction of relay transceivers the cost for whom to relay traffic is $c \in \{c : \mathcal{K}(c) = k\}$. Let η_0 be the fraction of receiving transceivers who has 0 tokens and hence, they cannot request relay service from other transceivers when relay transmissions are needed.

$$\eta_o = \sum_{i=0}^{\infty} w(i) \eta_i(0) \quad (8)$$

Therefore, the probability that the relay transmission successfully takes places is

$$E = (1 - \eta_d)(1 - \eta_o) \quad (9)$$

Because the network is dynamic, η_d, η_0 vary over time and are difficult to compute. However, we are able to explicitly derive the optimal token supply if the cost and the demand rate are homogeneous. By taking the homogeneous cost as the average cost for relaying traffic and the homogeneous demand rate as the average demand rate, we obtain a suboptimal token supply for the relay system while the complexity is significantly reduced. In the simulations, we will show the performance of this suboptimal choice of token supply compared to the optimal one.

A. Token holding distribution and optimal supply

For the homogeneous cost c , all transceivers use a same threshold strategy in all periods. As we know from the last section, there is a unique threshold $K = \mathcal{K}(c)$ strategy that the transceivers adopt. Therefore, no transceivers hold more than K tokens. Hence, there are two feasibility conditions that the token distribution must satisfy

$$\sum_{k=0}^K \eta(k) = 1, \quad \sum_{k=0}^K k\eta(k) = \alpha \quad (10)$$

Moreover, it is simply that $\eta_0 = \eta(0), \eta_d = \eta(K)$.

If the current token distribution is η and the transceivers follow the strategy with threshold K , the token distribution in the next time period can be calculated in a straightforward way.

$$\begin{aligned} \eta^+(0) &= \lambda(1 - \eta(K))\eta(1) + (1 + \lambda(\eta(0) - 1))\eta(0) \\ \eta^+(k) &= \lambda(1 - \eta(0))\eta(k-1) \\ &\quad + \lambda(1 - \eta(K))\eta(k+1) \\ &\quad + (1 + \lambda(\eta(0) + \eta(K) - 2))\eta(k), \quad 1 \leq k \leq K-1 \\ \eta^+(K) &= \lambda(1 - \eta(0))\eta(K-1) \\ &\quad + (1 + \lambda(\eta(K) - 1))\eta(K) \end{aligned} \quad (11)$$

The transceivers with k tokens in the next period consist of: (1) transceivers who have $k - 1$ tokens in the current period become the relay transceivers and get a token; (2) transceivers who have $k + 1$ tokens in the current period become the receiver transceivers and lose a token; (3) transceivers who have k tokens and do not get or lose tokens. If the token distribution remains the same in the next period, then we say that the token distribution is invariant, i.e. $\eta^+ = \eta$. The next proposition characterizes the invariant distribution by solving $\eta^+ = \eta$ according to (11).

Proposition 4: If all transceivers follow the same optimal threshold strategy with the threshold K , the invariant token holding distribution η is independent of the demand rate λ and satisfies,

$$\eta(k) = \left(\frac{1 - \eta(0)}{1 - \eta(K)} \right)^k \eta(0), \forall k = 0, 1, \dots, K \quad (12)$$

The invariant token distribution is a restricted geometric distribution which only depends on the threshold that transceivers use. There are no closed-form expressions of $\eta(0)$ and $\eta(K)$ and hence, we cannot derive the closed form expression for the system efficiency. Fortunately though, (12) provides sufficient information to find the optimal token supply to maximize the system efficiency.

Proposition 5: If all transceivers follow the same optimal threshold strategy with the threshold K , the token supply α that maximizes the system efficiency is $K/2$ per transceiver on average. Moreover, the maximum efficiency is

$$E = \left(1 - \frac{1}{K+1} \right)^2 \quad (13)$$

Proposition 5 proves that there is an optimal token supply for the relay network where transceivers use the same threshold strategy: it is not too small such that transceivers do not have enough tokens to make relay requests nor too large such that transceivers decide to not provide relay service when they are needed for relaying. For the system designer to efficiently operate the relay network, it needs to understand the transceivers' strategic behaviors and issue the appropriate number of tokens. This brings up another important problem for the system designer to determine the thresholds that the transceivers want to use because, as we know from Proposition 5, the optimal token supply is half the threshold per transceiver on average.

B. Determining the threshold

We already know that transceivers adopt threshold strategies to maximize their utilities in Section III. In this subsection, we provide an efficient algorithm to determine which threshold strategy the transceivers may want to use for different network conditions. One way to find out the threshold is to run a brute-force algorithm on all threshold strategies to check whether they are optimal for the given network conditions. A much more efficient way is to perform a bisection search. To do this, we need to establish an upper bound on the threshold such that the strategy is optimal.

Proposition 6: Given the network conditions λ, β, b, c , if a threshold strategy is optimal, then the threshold K is upper bounded by

$$K < \log_{\frac{1-\beta+\lambda\beta}{\lambda\beta}} \frac{1-\beta+\lambda\beta}{1-\beta+2\lambda\beta} \left(\frac{b}{c} + 1 \right) \triangleq \bar{K} \quad (14)$$

With the upper bound on the threshold, the bisection-based algorithm is constructed in Algorithm 1. When the threshold is determined, the system designer simply issues $K^*/2$ tokens per transceiver on average into the relay network. The induced efficiency has been proven optimal if the cost is homogeneous in the previous subsection. For the case that the cost is heterogeneous, we rely on numerical methods to investigate the performance in the next section.

ALGORITHM 1: Optimal Threshold Computation

Input: System parameters β, λ, b, c

Output: Optimal threshold K^*

Compute \bar{K} according to (14). Set $K^H = \bar{K}, K^L = 0$. ;

repeat

 Assign $K_{test} = (K^H + K^L)/2$;

 Compute the marginal utilities $M(K_{test} - 1)$ and $M(K_{test})$;

 If $M(K_{test} - 1) < c/\beta$, set $K^H = K_{test}$;

 IF $M(K_{test}) \geq c/\beta$, set $K^L = K_{test}$

until $M(K_{test} - 1) \geq c/\beta, M(K_{test}) < c/\beta$;

V. SIMULATIONS

In this section, we provide illustrative results to highlight the various design aspects of our proposed token framework. In the simulations, $N = 1500$ transceivers are distributed in a square area consisting of 100 smaller squares with size of $1\text{km} \times 1\text{km}$. At the center of each small square there is a fixed source transceiver (e.g. an access point or a base station). In each time period, each transceiver moves to a different location according to the random waypoint mobility model and needs to receive data from the source of the square that it belongs to. We consider path loss and shadow fading for the channel model [32], $P_{received} = P_{transmitted} - PL(d_0) - 10\alpha \log(d/d_0) - \chi$, where $PL(d_0)$ is the path loss of the reference distance d_0 , d is the distance between the source and the destination, α is the path loss factor, χ is a normally (Gaussian) distributed random variable representing the effect of shadow fading. We assume that the maximum transmission power of a transceiver is 15dBm, the bandwidth of a channel is 10MHz and the target data rate is 10Mbps. If by using the maximum power the target data rate cannot be achieved for a receiving transceiver, then a relay transmission is needed. Hence, using the above parameter values, the relay transmission demand rate is about $\lambda = 0.1$. The required relay transmission power is calculated using equation (2). The (normalized) cost c is the ratio of the relay transmission power P_r (in mW) to the expected benefit $\mathbb{E}b(r^{target})$ of achieving r^{target} .

Figure 3 provides an illustrative snapshot of a part of the network (9 square areas out of 100 squares) in one time period. The star nodes are the receiving transceivers that need the help of relay transmissions. We can see that these transceivers are

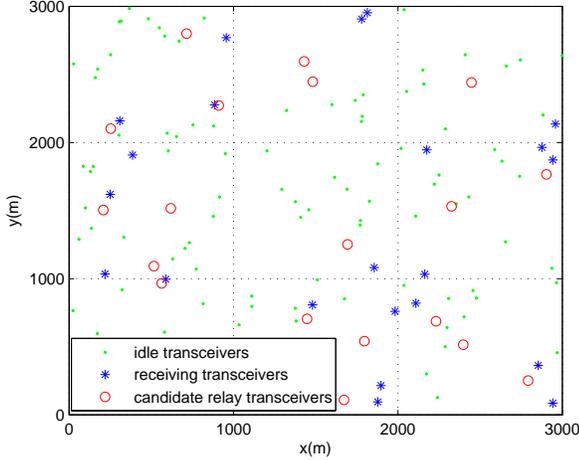


Fig. 3. Snapshot of a part of the simulated network.

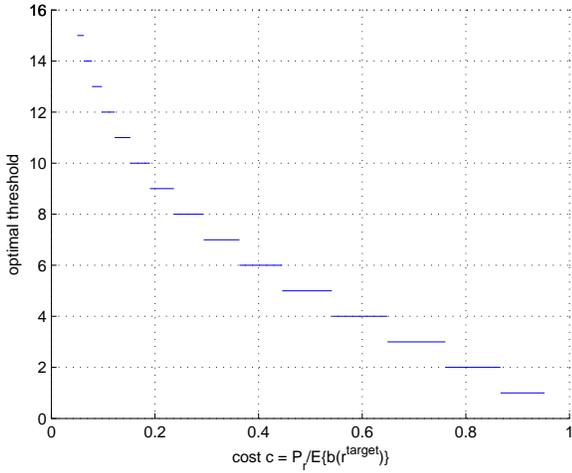


Fig. 4. Optimal threshold for various costs.

usually at the border of the squares where the received signal strength from the source transceiver is low. The circle nodes are the candidate relay transceivers selected based the lowest relay transmission power criterion using equation (2). These transceivers are usually between the source transceivers and the receiving transceivers. (Since there is the shadowing effect, this may not be always true.) The nodes represented by simple dots are the remaining transceivers.

Figure 4 illustrates the mapping from the normalized cost to the optimal threshold $\mathcal{K}(c)$ given. For each threshold k , there is a continuous cost interval that makes the corresponding threshold strategy σ_k optimal. For all costs that are less than $c_0 = 0.952$, the optimal threshold is unique for each cost. Moreover if $c \leq c'$, then $\mathcal{K}(c) \geq \mathcal{K}(c')$. This means that the transceivers are more willing to provide the relaying service if its required relay transmission power is lower.

In this set of simulations, we investigate the impact of the token supply on the system efficiency, i.e. the probability that the relay transmission successfully takes place. With the relay selection criterion being the least relay transmission power, the average required relay transmission power is about 10dBm. Hence, the average normalized cost is about $\bar{c} = 0.1$.

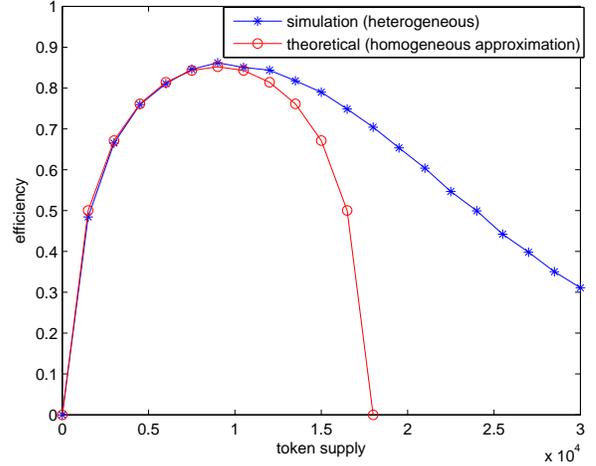


Fig. 5. Optimal token supply.

According to Figure 4, the optimal threshold is $K^{opt} = 12$ for $\bar{c} = 0.1$. If we use this average normalized cost to derive the suboptimal token supply, then the optimal token supply is $\alpha = K^{opt}/2 = 6$ tokens per transceiver on average which corresponds to $W = 9000$ tokens in total. In Figure 5, we vary the number of tokens issued in the system and compare the simulated system efficiency (computed using the actual heterogeneous costs) to the theoretical system efficiency (computed using the homogeneous cost as the average cost). Several observations are notable. First, we can see from these experiments that the optimal token supply is very close to the derived suboptimal token supply ($W = 9000$ using the homogeneous approximation). Second, when the token supply is small, the theoretical system efficiency using the homogenous cost approximation is very close to the true simulated efficiency with heterogeneous costs. Third, when the token supply is large, the true simulated efficiency exhibits a long-tail effect compared to the theoretical estimation. This is because in the homogeneous cost scenario, if the supplied tokens are more than the threshold (in our case, more than $12 \times 1500 = 18000$ tokens), then no transceiver will ever want to provide the relaying service and hence, the efficiency is 0. However, in the heterogeneous cost case, sometimes transceivers may want to use very high thresholds if the cost is very small and hence, even with a very large token supply, there still is a probability that relay transmissions take place. We can expect that as the costs become more heterogeneous, the long-tail effect is more obvious.

Next, we illustrate the throughput improvement by adopting the relay transmission among mobile transceivers. Specifically, we compare the data rate achieved by the relay transmission with that achieved by the direct transmission. Since there are multiple simultaneous relay transmissions going on in the system, the co-channel interference may have a significant impact on the throughput performance. Therefore, two interference models are considered to quantify this impact: perfect orthogonal channels and random channel assignment for various numbers of available orthogonal channels. In the

Interference Model	Random Channel Assignment					Orthogonal Channels
	1 CH	2 CHs	3 CHs	4 CHs	5 CHs	
Data Rate (Mbps)	7.13	8.37	8.89	9.13	9.3	10

TABLE II
ACHIEVED DATA RATE FOR VARIOUS INTERFERENCE MODELS.

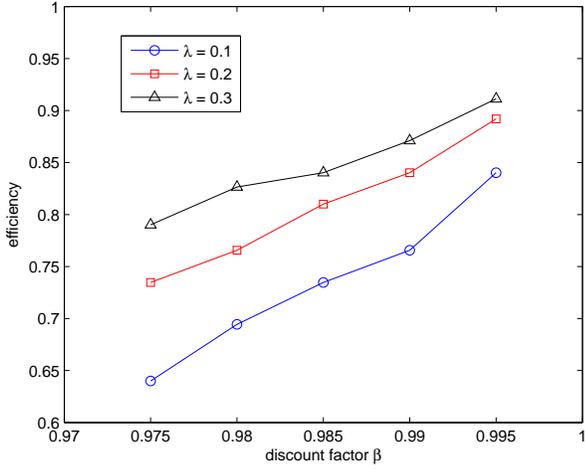


Fig. 6. Impact of the discount factor and the relay demand rate on system efficiency.

perfect orthogonal channels model, all relay transmissions take place on orthogonal channels. In the random channel assignment, each relay transmission randomly picks one channel from a number of orthogonal channels. Our simulation shows that the average data rate of the direct transmission is 5.8 Mbps and the achieved data rates under various interference models are shown in Table II. Hence, the relay transmission significantly improves the throughput. Even if we consider co-channel interference between relay transmissions, using a small number of channels is able to achieve performance that is close to that by perfectly orthogonal channels. Note that we used a very simple channel assignment scheme and other advanced channel assignment schemes may be able to achieve even better performance.

In the next set of simulations, we investigate the impact of the discount factor β and the relay transmission demand rate λ on the system performance. Figure 6 shows that the system efficiency is increasing with the discount factor and the relay transmission demand. This is because tokens become more valuable and transceivers tend to use strategies with higher thresholds in order to accumulate more tokens. Hence, the system designer can supply a larger amount of tokens that leads to both a lower probability that transceivers cannot request the relaying service and a lower probability that relays do not provide the relaying service. Therefore, the mobile relay technique is more helpful when the relay transmission demand is higher and transceivers stay in the system for a longer time.

Finally, we investigate the performance of our token system design in scenarios where transceivers have heterogeneous relay demand probabilities. The relay demand probability in this simulation is uniformly distributed between [0.05 0.15] with the mean relay demand probability of $\bar{\lambda} = 0.1$. In Table III, we report the simulated and theoretically estimated system

Token Supply	4500	6000	7500	9000	10500	12000	13500	
Efficiency	Sim.	0.72	0.80	0.83	0.86	0.85	0.83	0.81
	Est.	0.76	0.81	0.84	0.85	0.84	0.81	0.76

TABLE III
SYSTEM EFFICIENCY FOR HETEROGENEOUS RELAY DEMAND PROBABILITIES.

efficiency by deploying various numbers of tokens. It is shown that the estimated efficiency using the mean relay demand probability is close to the simulated efficiency. Moreover, the derived optimal token supply for the homogeneous relay demand probability scenario corresponds to that when the relay demand probabilities are heterogeneous for transceivers.

VI. CONCLUSION

In this paper, we propose a novel mechanism for providing self-interested transceivers with incentives to relay traffic for other wireless transceivers using a token system. The design of the token system is formulated as a two-level optimization problem where the transceiver-level problem determines the transceivers' optimal strategy and the designer-level problem determines the optimal token supply in the network. Importantly, in this paper, we rigorously characterize the structural properties exhibited by the optimal strategies adopted by the transceivers and prove that they are threshold strategies. We also formally characterize the relation between the thresholds and the network parameters, such as the relay transmission cost. This threshold property allows a better understanding of transceivers' strategic behaviors when facing different costs. The token supply was often a neglected parameter when designing similar token systems in existing literature. Our findings in this paper emphasize that the token supply represents a critical design parameter affecting the system efficiency.

APPENDIX A PROOF OF LEMMA

For $k \leq K_\sigma$, the value functions are

$$\begin{aligned}
 V(0) &= (1 - \lambda)\beta V(0) + \lambda(-c + \beta V(1)) \\
 V(k) &= (1 - 2\lambda)\beta V(k) + \lambda(-c + \beta V(k+1)) \\
 &\quad + \lambda(b + \beta V(k-1)) \\
 V(K_\sigma) &= (1 - \lambda)\beta V(K_\sigma) + \lambda(b + \beta V(K_\sigma - 1))
 \end{aligned} \tag{15}$$

These are second-order homogeneous difference equations. However, the solution expression is complicated and does not provide any direct results of the property of the marginal utilities. Therefore, we study these equations using an alternative way. Rearranging the terms and replacing with $M(k), \forall k \leq K_\sigma$ yields

$$\Phi M = u \tag{16}$$

where

$$\Phi = \begin{bmatrix} \phi_1 & \phi_2 & 0 & \cdots & 0 \\ \phi_2 & \phi_1 & \phi_2 & \ddots & \vdots \\ 0 & \phi_2 & \phi_1 & \phi_2 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ 0 & \cdots & 0 & \phi_2 & \phi_1 \end{bmatrix}, u = \begin{bmatrix} \lambda b \\ 0 \\ \vdots \\ 0 \\ \lambda c \end{bmatrix} \tag{17}$$

and $\phi_1 = 1 - (1 - 2\lambda)\beta, \phi_2 = -\lambda\beta$.

(1) Suppose $\exists k^* \leq K_\sigma - 2$, such that $M(k^*) \leq 0$. We first show that neither $M(k^* - 1)$ nor $M(k^* + 1)$ is non-positive and then show they also cannot be positive. With this, we find a contradiction to conclude that $M(k) > 0, \forall k \leq K_\sigma$.

For “neither $M(k^* - 1)$ nor $M(k^* + 1)$ is non-positive”, we study two cases.

Case 1: $0 \geq M(k^* - 1) \geq M(k^*)$ (and $0 \geq M(k^*) \geq M(k^* + 1)$). By $\Phi M = u$,

$$\begin{aligned} M(k^* + 1) &= \frac{\phi_2 M(k^* - 1) + \phi_1 M(k^*)}{-\phi_2} \\ &\leq \frac{(\phi_1 + \phi_2) M(k^*)}{-\phi_2} \leq M(k^*) \end{aligned} \quad (18)$$

Recursively, it should be $M(K_\sigma - 1) \leq M(K_\sigma - 2) \leq \dots \leq M(k^*) \leq M(k^* - 1) \leq 0$. However, it is not true because otherwise we will get

$$\begin{aligned} \phi_1 M(K_{\sigma_1} - 1) &= \lambda c - \phi_2 M(K_\sigma - 2) \\ &> -\phi_2 M(K_\sigma - 2) \geq -\phi_2 M(K_\sigma - 1) \end{aligned} \quad (19)$$

Therefore, $\phi_1 < -\phi_2$. This is a contradiction. With similar arguments, $0 \geq M(k^*) \geq M(k^* + 1)$ is not true.

Case 2: $0 \geq M(k^*) \geq M(k^* - 1)$ (and $0 \geq M(k^* + 1) \geq M(k^*)$). This case is similar to the first case except that we go in the other direction of . The above two cases exclude the possibility that either $M(k^* + 1)$ or $M(k^* - 1)$ can be non-positive.

For “neither $M(k^* - 1)$ nor $M(k^* + 1)$ is positive”, it is obviously not true because otherwise

$$\phi_2 M(k^* - 1) + \phi_1 M(k^*) + \phi_2 M(k^* + 1) < 0 \quad (20)$$

This completes the proof for the first part of Lemma.

(2) It is sufficient to prove there does not exist any $k^* \leq K_\sigma - 2$, such that

$$M(k^* - 1) \leq M(k^*) \geq M(k^* + 1) \quad (21)$$

Suppose this is true, then

$$\begin{aligned} M(k) &= \frac{-\phi_2 M(k-1) - \phi_2 M(k+1)}{\phi_1} \\ &\leq \frac{-2\phi_2}{\phi_1} M(k) < M(k) \end{aligned} \quad (22)$$

which is a contradiction.

(3) By the second part of this lemma, if $M(k)$ is not a decreasing sequence, then it must be $M(K_\sigma - 1) \geq M(K_\sigma - 2)$. This is not true because otherwise

$$\begin{aligned} \lambda c &= \phi_2 M(K_\sigma - 2) + \phi_1 M(K_\sigma - 1) \geq \\ &(\phi_2 + \phi_1) M(K_\sigma - 1) \geq (\phi_2 + \phi_1) \frac{c}{\beta} > \lambda c \end{aligned} \quad (23)$$

which is a contradiction. Therefore $M(k)$ is a decreasing sequence.

APPENDIX B PROOF OF PROPOSITION 1

We instead show that a non-threshold strategy is not optimal. For a non-threshold strategy, there must exist $K_1, K_2 (K_2 > K_1)$ such that

$$\begin{aligned} \sigma(k) &= 1, \forall k < K_1 \\ \sigma(k) &= 0, \forall K_1 \leq k < K_2 \\ \sigma(k) &= 1, k = K_2 \end{aligned} \quad (24)$$

For the strategy to be optimal, we also need to check whether $M(K_2 - 1) \leq c/\beta, M(K_2) \geq c/\beta$ hold. We then show that these two conditions do not hold at the same time. In particular, we prove that if $M(K_2) \geq c/\beta$, then it must be $M(K_2 - 1) > c/\beta$ following the value equations given by (15).

We discuss the following two cases.

Case 1: $\sigma(K_2 + 1) = 0$. We have the following relation,

$$-\lambda\beta M(K_2 - 1) + (1 - (1 - 2\lambda)\beta) M(K_2) = \lambda c \quad (25)$$

which further yields

$$\begin{aligned} \lambda\beta M(K_2 - 1) &= (1 - (1 - 2\lambda)\beta) M(K_2) - \lambda c \\ &\geq (1 - (1 - 2\lambda)\beta) c/\beta - \lambda c = (1 - \beta) c/\beta + \lambda c > \lambda c \end{aligned} \quad (26)$$

Therefore, $M(K_2 - 1) > c/\beta$.

Case 2: $\sigma(K_2 + 1) = 1$. And there exists $K_3 > K_2$ such that $\forall K_2 \leq k < K_3, \sigma(k) = 1$. Then following similar arguments in Lemma, we have

$$M(K_2) > M(K_2 + 1) > \dots > M(K_3 - 1) \geq c/\beta \quad (27)$$

Because of the relation

$$\phi_2 M(K_2 - 1) + \phi_1 M(K_2) + \phi_2 M(K_2 + 1) = 0 \quad (28)$$

The marginal utility $M(K_2 - 1)$ satisfies

$$\begin{aligned} \lambda\beta M(K_2 - 1) &= (1 - (1 - 2\lambda)\beta) M(K_2) - \lambda\beta M(K_2 + 1) \\ &> (1 - (1 - \lambda)\beta) M(K_2) \geq (1 - (1 - \lambda)\beta) c/\beta > \lambda c \end{aligned} \quad (29)$$

Therefore $M(K_2 - 1) > c/\beta$. This completes the proof.

APPENDIX C PROOF OF PROPOSITION 2

Denote $M(k|c)$ as the marginal utility of holding k tokens when the cost is c . We will use the following result: For given λ, β, b and a threshold strategy σ_K with threshold K , the normalized marginal utility $M(k|c)/c$ decreases with the cost.

To prove this, consider two costs $c_1 > c_2$. Using the marginal utility equation (16),

$$\begin{aligned} \Phi(M_2/c_2 - M_1/c_1) &= u_2/c_2 - u_1/c_1 \\ &= (\lambda(b/c_2 - b/c_1) \ 0 \ \dots \ 0)^T \end{aligned} \quad (30)$$

By Lemma part (1),

$$M_2/c_2 - M_1/c_1 \succ 0 \quad (31)$$

This is the result that we need.

Write the normalized marginal utility by \tilde{M} .

(1) We first prove that there exists c^H , such that $\forall c \leq c^H, \tilde{M}(K - 1) \geq 1/\beta$. By Lemma 2, $F(c) = \tilde{M}(K - 1|c) - 1/\beta$ is a decreasing function in c . We check the signs of $F(b)$ and $F(0)$ in the following.

For $c = b$, suppose $F(b) \geq 0$. By Lemma, $\tilde{M}(k|c) \geq 1/\beta, \forall k \leq K - 1$. However, this is not true because otherwise

$$\begin{aligned} &\lambda(1 + b/c) \\ &= (1 - (1 - \lambda)\beta) \tilde{M}(0|c) \\ &+ (1 - \beta) \sum_{k=1}^{K-2} \tilde{M}(k|c) \\ &+ (1 - (1 - \lambda)\beta) \tilde{M}(K - 1|c) \\ &< K(1 - \beta) M + 2\lambda\beta M \leq \infty \end{aligned} \quad (32)$$

which is a contradiction. Therefore, there exists a unique c^H such that for all $c < c^H$, $\tilde{M}(K-1) \geq 1/\beta$.

(2) Next we prove that there exists a $c^L < c^H$, such that $\forall c > c^L$, $\tilde{M}(K) < 1/\beta$. According to (16), it is equivalent to show that

$$\tilde{M}(K-1) < \frac{\phi_1 + \phi_2}{-\phi_2} \frac{1}{\beta} \quad (33)$$

Write

$$G(c) = \tilde{M}(K-1|c) - \frac{\phi_1 + \phi_2}{-\phi_2} \frac{1}{\beta} \quad (34)$$

$G(c)$ is decreasing in c . We check the signs of $G(c^H)$ and $G(0)$. For $c = c^H$. It is easy to see $G(c^H) < 0$. For $c \rightarrow 0$, with similar arguments for finding c^H , $G(c \rightarrow 0) > 0$. Therefore, there exists a unique c^L such that for all $c > c^L$, $\tilde{M}(K) < 1/\beta$. Combining both parts, we complete the proof.

APPENDIX D PROOF OF PROPOSITION 3

According to Proposition 2, for any threshold strategy, there is a continuous interval of c such that it is optimal, it is sufficient to prove that the intervals overlap for two consecutive threshold strategies. In particular, we only need to prove that for two consecutive thresholds $K_1, K_2 (K_2 = K_1 + 1)$, and the corresponding cost intervals $(c_1^L, c_1^H], (c_2^L, c_2^H]$, the following holds: $c_1^L = c_2^H$.

We need to show that if $c = c_2^H$, the strategy with the threshold K_1 must have

$$M_1(K_1 - 1) = \frac{\phi_1 + \phi_2}{-\phi_2} \frac{c_2^H}{\beta} \quad (35)$$

Because $M_2(K_2 - 1) = c_2^H/\beta$, and we use this to eliminate the last row in (17). The coefficient matrix reduces by 1 and is identical to that for K_1 . Moreover, the right-hand side is also identical to what we need to solve. Therefore,

$$\begin{aligned} M_1(K_1 - 1) &= M_2(K_2 - 2) \\ &= \frac{\phi_1 M_2(K_2 - 1) - \lambda c_2^H}{-\phi_2} = \frac{\phi_1 + \phi_2}{-\phi_2} \frac{c_2^H}{\beta} \end{aligned} \quad (36)$$

Therefore $c_1^L = c_2^H$. And c_0 is chosen as the upper boundary value for $K = 1$.

APPENDIX E PROOF OF PROPOSITION 5

It is convenient to first solve the following maximization problem

$$\begin{aligned} &\text{maximize} && (1-x_1)(1-x_2) = 1-x_1-x_2+x_1x_2 \\ &\text{subject to} && x_1(1-x_1)^K = x_2(1-x_2)^K \\ &&& 0 \leq x_1, x_2 \leq 1 \end{aligned} \quad (37)$$

To solve this problem, set $f(x) = x(1-x)^K$, a straightforward calculus exercise shows that if $0 \leq x_1 \leq 1/(K+1) \leq x_2 \leq 1$ and $f(x_1) = f(x_2)$ then,

(a) $x_1 + x_2 \geq 1/(K+1)$ with equality achieved only at $x_1 = x_2 = 1/(K+1)$.

(b) $x_1 x_2 \leq 1/(K+1)$ with equality achieved only at $x_1 = x_2 = 1/(K+1)$.

Putting (a) and (b) together shows that the optimal solution to the maximization problem is to have $x_1 = x_2 = 1/(K+1)$ and the maximized objective function value is

$$\max(1-x_1)(1-x_2) = \left(1 - \frac{1}{K+1}\right)^2 \quad (38)$$

Now consider the threshold K strategy and let η be the corresponding invariant distribution. If we take $x_1 = \eta_0, x_2 = \eta_d$ then our characterization of the invariant distribution shows that $f(x_1) = f(x_2)$. By definition, $E = (1-x_1)(1-x_2)$ so

$$E = \left(1 - \frac{1}{K+1}\right)^2 \quad (39)$$

Taken together, these are the assertions which were to be proved.

APPENDIX F PROOF OF PROPOSITION 6

If the strategy is optimal, simple algebra and induction on (17) show that the marginal utilities satisfy

$$M(k) \geq \left(\frac{1 + (\lambda - 1)\beta}{\lambda\beta}\right)^{K-1-k} \frac{c}{\beta} \quad (40)$$

Because

$$\begin{aligned} \lambda(b+c) &= \lambda\beta(M(0) + M(K-1)) \\ &\quad + (1-\beta)(1+1/K) \sum_{k=0}^{K-1} M(k) \end{aligned} \quad (41)$$

Substitute (40) into (41), we establish the desired upper bound.

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