

Linearly Coupled Communication Games

Yi Su and Mihaela van der Schaar
Department of Electrical Engineering, UCLA

Abstract—This paper discusses a special type of multi-user communication scenario, in which users’ utilities are linearly impacted by their competitors’ actions. We explicitly characterize the Nash equilibrium and Pareto boundary of the achievable utility region. To improve the performance in the non-cooperative scenarios, we investigate the properties of an alternative solution concept named conjectural equilibrium, in which individual users compensate for their lack of information by forming internal beliefs about their competitors. The global convergence of the best response and Jacobi update dynamics that achieve various conjectural equilibria are analyzed. It is shown that the Pareto boundaries of the investigated linearly coupled games can be sustained as stable conjectural equilibria if the belief functions are properly initialized. The investigated models apply to a variety of realistic applications encountered in the multiple access design, including wireless random access and flow control.

I. INTRODUCTION

Game theory provides a formal framework for studying the interactions of strategic agents. Recently, there has been a surge in research activities that employ game theory to model and analyze a wide range of application scenarios in modern communication networks [1]- [4]. In communication networks, any action taken by a single user usually affects the utilities of the other users sharing the same resources. Depending on the characteristics of different applications, numerous game-theoretical models and solution concepts have been proposed to describe the multi-user interactions and optimize the users’ decisions in communication networks. Roughly speaking, the existing multi-user research can be categorized into two types, non-cooperative games and cooperative games. Various game theoretic solutions were developed to characterize the resulting performance of the multi-user interaction, including the Nash Equilibrium (NE) and the Pareto-optimality [18].

Non-cooperative approaches generally assume that the participating users simply choose actions to selfishly maximize their individual utility functions. It is well-known that if devices operate in a non-cooperative manner, this will generally limit their performance as well as that of the whole system, because the available resources are not always efficiently exploited due to the conflicts of interest occurring among users [5]. Most non-cooperative approaches are devoted to investigating the existence and properties of the NE. In particular, several non-cooperative game models, such as S-modular games, congestion games, and potential games, have been extensively applied in various communication scenarios [6]- [9]. The price of anarchy, a measure of how good the system performance is when users play selfishly and reach the NE instead of playing to achieve the social optimum, has also been addressed in several communication network applications

[10] [11].

On the other hand, cooperative approaches in communication theory usually focus on studying how users can jointly improve their performance when they cooperate. For example, the users may optimize a common objective function, which represents the Pareto-optimal social welfare allocation rule based on which the system-wide resource allocation is performed [12] [13]. A profile of actions is Pareto-optimal if there is no other profile of actions that makes every player at least as well off and at least one player strictly better off. Allocation rules, e.g. network utility maximization, can provide reasonable allocation outcomes by considering the trade-off between fairness and efficiency. Most cooperative approaches focus on studying how to efficiently find the optimum joint policy. It is worth mentioning that information exchanges among users is generally required to enable users to coordinate in order to achieve and sustain Pareto-efficient outcomes.

In this paper, we present a game model for a particular type of non-cooperative multi-user communication scenario. We name it linearly coupled communication games, because users’ utilities are linearly impacted by their competitors’ actions. In particular, the main contributions of this paper are as follows. First, based on the assumptions that we make about the properties of users’ utility, we characterize the inherent structures of the utility functions for the linearly coupled games. Furthermore, based on the derived utility forms, we explicitly quantify the NE and Pareto boundary for the linearly coupled communication games. The price of anarchy incurred by the selfish users playing the Nash strategy is quantified. In addition, to improve the performance in the non-cooperative scenarios, we investigate an alternative solution: conjectural equilibrium (CE). Using this approach, individual users are modeled as belief-forming agents that develop internal beliefs about their competitors and behave optimally with respect to their individual beliefs. Necessary and sufficient conditions that guarantee the convergence of different dynamic update mechanisms, including the best response and Jacobi update, are addressed. We prove that these adjustment processes based on conjectures and non-cooperative individual optimization can be globally driven to Pareto-optimality in the linearly coupled games without the need of real-time coordination information exchange among agents.

The rest of this paper is organized as follows. Section II defines the linearly coupled communication games. For the investigated game models, Section III explicitly computes the NE and Pareto boundary of the achievable utility region and quantifies the price of anarchy. Section IV introduces the CE

and investigates its properties under both the best response and Jacobi update dynamics. Conclusions are drawn in Section V. Due to space limitations, the formal proofs are omitted; for these proofs, the reader is referred to [26].

II. GAME MODEL

In this section, we first provide a general game-theoretic formulation of the multi-user interaction in communication systems. Following the proposed definition, we define the linearly coupled communication games and provide concrete examples of the investigated game model.

A. Linearly Coupled Communication Games

The multi-user game in various communication scenarios can be formally defined as a tuple $\Gamma = \langle \mathcal{N}, \mathcal{A}, u, \mathcal{S}, s \rangle$. In particular, $\mathcal{N} = \{1, 2, \dots, N\}$ is the set of communication devices, which are the rational decision-makers in the system. Define \mathcal{A} to be the joint action space $\mathcal{A} = \times_{n \in \mathcal{N}} \mathcal{A}_n$, with \mathcal{A}_n being the action set available for user n . As opposed to the traditional strategic game definition [18], we introduce two new elements \mathcal{S} and s into the game formulation. Specifically, \mathcal{S} is the *state space* $\mathcal{S} = \times_{n \in \mathcal{N}} \mathcal{S}_n$, where $\mathcal{S}_n \subseteq \mathcal{R}_+$ is the part of the state relevant to user n . The state is defined to capture the effects of the multi-user coupling such that each user's utility solely depends on its own state and action. In other words, the utility function $u = \times_{n \in \mathcal{N}} u_n$ is a mapping from the individual users' state space and action space to real numbers, $u_n : \mathcal{S}_n \times \mathcal{A}_n \rightarrow \mathcal{R}$. The state determination function $s = \times_{n \in \mathcal{N}} s_n$ maps joint actions to states for each component $s_n : \mathcal{A} \rightarrow \mathcal{S}_n$. To capture the performance tradeoff, the utility region is defined as $\mathcal{U} = \{(u_1(\mathbf{a}), \dots, u_N(\mathbf{a})) \mid \exists \mathbf{a} = (a_1, a_2, \dots, a_N) \in \mathcal{A}\}$.

Definition 1: A multi-user interaction is considered a *linearly coupled communication game* if the action set $\mathcal{A}_n \subseteq \mathcal{R}_+$ is convex and the utility function u_n satisfies:

$$u_n(\mathbf{a}) = a_n^{\beta_n} \cdot s_n(\mathbf{a}), \quad (1)$$

in which $\beta_n > 0$. In particular, the basic assumptions about $s_n(\mathbf{a})$ include:

A1: $s_n(\mathbf{a})$ is non-negative;

A2: Denote $s'_{nm}(\mathbf{a}) = \frac{\partial s_n(\mathbf{a})}{\partial a_m}$ and $s''_{nm}(\mathbf{a}) = \frac{\partial^2 s_n(\mathbf{a})}{\partial a_m^2}$. $s_n(\mathbf{a})$ is strictly linear decreasing in $a_m, \forall m \neq n$, i.e. $s'_{nm}(\mathbf{a}) < 0$ and $s''_{nm}(\mathbf{a}) = 0$; $s_n(\mathbf{a})$ is non-increasing and linear in a_n , i.e. $s'_{nn}(\mathbf{a}) \leq 0$ and $s''_{nn}(\mathbf{a}) = 0$.

A3: $\frac{s_n(\mathbf{a})}{s'_{nm}(\mathbf{a})}$ is an affine function, $\forall n \in \mathcal{N} \setminus \{m\}$.

A4: $\frac{s'_{nm}(\mathbf{a})}{s_n(\mathbf{a})} = \frac{s'_{km}(\mathbf{a})}{s_k(\mathbf{a})}, \forall n, k \in \mathcal{N} \setminus \{m\}$; $\frac{s'_{mm}(\mathbf{a})}{s_m(\mathbf{a})} = 0$ or $\frac{s'_{nm}(\mathbf{a})}{s_n(\mathbf{a})}, \forall n \neq m$.

Assumptions A1 and A2 indicate that increasing a_m for any $m \neq n$ within the domain of $s_n(\mathbf{a})$ will linearly decrease user n 's utility. Assumptions A3 and A4 imply that a user's action has proportionally the same impact over the other users' utility. In [26], we investigate the structure of the utility functions that satisfy assumptions A1-A4 and find that there are two basic types of linearly coupled games satisfying the assumptions A1-A4.

In Type I games, user k 's action linearly decreases all the users' states but itself. The utility functions take the form

$$u_n(\mathbf{a}) = a_n^{\beta_n} \cdot \prod_{m \neq n} (\mu_m - \tau_m a_m). \quad (2)$$

For example, in the random access networks [15], the action of a node is to select its transmission probability and a node n will attempt transmission of a packet with transmit probability p_n . The action set available to node n is $\mathcal{A}_n = [0, 1]$ for all $n \in \mathcal{N}$. In this case, the utility function is defined as

$$u_n(\mathbf{p}) = p_n \cdot \prod_{m \neq n} (1 - p_m). \quad (3)$$

In Type II games, all the users share the same state function and their utility functions are given by

$$u_n(\mathbf{a}) = a_n^{\beta_n} \cdot (\mu - \sum_{m=1}^N \tau_m a_m). \quad (4)$$

For example, in flow control [16], N Poisson streams of packets are serviced by a single exponential server with departure rate μ and each class can adjust its throughput r_n . The utility function is defined as the weighted ratio of the throughput over the average experienced delay:

$$u_n(\mathbf{r}) = r_n^{\beta_n} \cdot (\mu - \sum_{m=1}^N r_m), \quad (5)$$

in which $\beta_n > 0$ is interpreted as the weighting factor. Specifically, we can see that the state determination functions are $s_n(\mathbf{p}) = \prod_{m \in \mathcal{N} \setminus \{n\}} (1 - p_m)$ in (3) and $s_n(\mathbf{r}) = \mu - \sum_{m=1}^N r_m$ in (5).

For linearly coupled communication games, we want to compare the performance attained by different game-theoretic solution concepts. On one hand, it is well-known that NE is generally inefficient [17], but it may not require explicit message exchanges, while Pareto-optimality can usually be achieved by exchanging explicit coordination messages among the participating users. On the other hand, in several recent works [14] [15], we have applied an alternative solution in different communication scenarios to improve the system performance in non-cooperative settings, namely the conjectural equilibrium [20]. The following sections aim to compare the solutions of NE, Pareto boundary, and CE in terms of the payoffs and informational requirements in the linearly coupled multi-user interaction satisfying the assumptions A1-A4. A brief summary of the properties of Type I games will be provided in Section IV-E. For the details about various game-theoretic solutions for Type I games, we refer the readers to [15]. The rest of this paper will focus on Type II games.

III. COMPUTATION OF THE NASH EQUILIBRIUM AND PARETO BOUNDARY IN TYPE II GAMES

Definition 2: A profile \mathbf{a} of actions is a *Nash equilibrium* of Γ if $u_n(a_n, \mathbf{a}_{-n}) \geq u_n(a'_n, \mathbf{a}_{-n})$ for all $a'_n \in \mathcal{A}_n$ and $n \in \mathcal{N}$.

At NE of the Type II games, we have $\frac{\partial u_n(\mathbf{a})}{\partial a_n} = 0, \forall n \in \mathcal{N}$. It is equivalent to solving the following linear equations

$$(1 + \beta_n)\tau_n a_n + \beta_n \sum_{m \neq n} \tau_m a_m = \beta_n \mu, \forall n \in \mathcal{N}. \quad (6)$$

The closed-form solution has been addressed in [21] for $\tau_n = 1, \forall n \in \mathcal{N}$. For the general case, we can verify that the NE is

$$a_n^{NE} = \frac{\beta_n \mu}{\tau_n (1 + \sum_{m=1}^N \beta_m)}, \forall n \in \mathcal{N}. \quad (7)$$

Since $\log(\cdot)$ is concave and $\log[u_n(\mathbf{a})]$ is a composition of affine functions [19], $u_n(\mathbf{a})$ is log-concave in \mathbf{a} and the log-utility region $\log \mathcal{U}$ is convex. Therefore, we can characterize the Pareto boundary of the utility region as a set of \mathbf{a} optimizing the following weighted proportional fairness objective:

$$\max_{\mathbf{a}} \sum_{n=1}^N \omega_n \log[u_n(\mathbf{a})], \quad (8)$$

for all possible sets of $\{\omega_n\}$ satisfying $\omega_n \geq 0$ and $\sum_{n=1}^N \omega_n = 1$. Its optimal solution is given by

$$a_n^{PB} = \frac{\omega_n \beta_n \mu}{\tau_n (1 + \sum_{m=1}^N \omega_m \beta_m)}, \forall n \in \mathcal{N}. \quad (9)$$

Based on Equations (7) and (9), we can see that, due to the lack of coordination, the NE in Type II games is always strictly Pareto inefficient. In the next section, we will investigate how the non-cooperative CE solution can improve the system performance for Type II games.

IV. CONJECTURAL EQUILIBRIUM FOR THE LINEARLY COUPLED GAMES

A. Definitions

In game-theoretic analysis, conclusions about the reached equilibria are based on assumptions about what knowledge the players possess. For example, the standard NE strategy assumes that every player believes that the other players' actions will not change at NE. Therefore, it chooses to myopically maximize its immediate payoff [18]. Therefore, the players operating at equilibrium can be viewed as decision makers behaving optimally with respect to their *beliefs* about the strategies of other players.

To avoid detrimental Nash strategy and encourage cooperation, the conjecture-based model has been introduced in [20] to enable non-cooperative players to build belief models about how their competitors' reactions vary in response to their own action changes. Specifically, each player has some belief about the state that would result from performing its available actions. The *belief function* \tilde{s}_n is defined to be $\tilde{s}_n : \mathcal{A}_n \rightarrow \mathcal{S}_n$ such that $\tilde{s}_n(a_n)$ represents the state that player n believes it would result in if it selects action a_n . Notice that the beliefs are not expressed in terms of other players' actions and preferences, and the multi-user coupling in these beliefs is captured indirectly by individual players forming conjectures of the effects of their own actions. By deploying such a behavior model, players will no longer adopt myopic behaviors

that do not forecast \tilde{s}_n , but rather they will form beliefs $\tilde{s}_n(a_n)$ about how their actions a_n will influence the aggregate effects \tilde{s}_n incurred by their competitors' responses and, based on these beliefs, they will choose the action $a_n \in \mathcal{A}_n$ if it believes that this action will maximize its utility. The steady state of such a play among belief-forming agents can be characterized as a conjectural equilibria.

Definition 3: In the game Γ , a configuration of belief functions $(\tilde{s}_1^*, \dots, \tilde{s}_N^*)$ and a joint action $a^* = (a_1^*, \dots, a_N^*)$ constitute a conjectural equilibrium, if for each $n \in \mathcal{N}$,

$$\tilde{s}_n^*(a_n^*) = s_n(a_1^*, \dots, a_N^*) \text{ and } a_n^* = \arg \max_{a_n \in \mathcal{A}_n} u_n(\tilde{s}_n^*(a_n), a_n).$$

From the definition, we can see that, at CE, all players' expectations based on their beliefs are realized and each agent behaves optimally according to its expectation. In other words, agents' beliefs are consistent with the outcome of the play and they use "conjectured best responses" in their individual optimization program. The key challenges are how to configure the belief functions such that cooperation can be sustained in such a non-cooperative setting and how to design the evolution rules such that the communication system can dynamically converge to a CE having satisfactory performance.

B. Linear Beliefs

As discussed before, the belief functions need to be defined in order to investigate the existence of CE. To define the belief functions, we need to express agent n 's expected state \tilde{s}_n as a function of its own action a_n . The simplest approach is to design linear belief models for each user, i.e. player n 's belief function takes the form

$$\tilde{s}_n(a_n) = \bar{s}_n - \lambda_n(a_n - \bar{a}_n), \quad (10)$$

for $n \in \mathcal{N}$. The values of \bar{s}_n and \bar{a}_n are specific states and actions, called *reference points* and λ_n is a positive scalar. In other words, user n assumes that other players will observe its deviation from its reference point \bar{a}_n and the aggregate state deviates from the reference point \bar{s}_n by a quantity proportional to the deviation of $a_n - \bar{a}_n$. How to configure \bar{s}_n, \bar{a}_n , and λ_n will be addressed in the rest of this paper. We focus on the linear belief represented in (10), because this simple belief form is sufficient to drive the resulting non-cooperative equilibrium to the Pareto boundary.

The goal of user n is to maximize its expected utility $a_n^{\beta_n} \cdot \tilde{s}_n(a_n)$ taking into account the conjectures that it has made about the other users. Therefore, the optimization a user needs to solve becomes:

$$\max_{a_n \in \mathcal{A}_n} a_n^{\beta_n} \cdot [\bar{s}_n - \lambda_n(a_n - \bar{a}_n)]. \quad (11)$$

For $\lambda_k > 0$, user n believes that increasing a_n will further reduce its conjectured state \bar{s}_n . The optimal solution of (11) is given by

$$a_n^* = \frac{\beta_n(\bar{s}_n + \lambda_n \bar{a}_n)}{\lambda_n(1 + \beta_n)}. \quad (12)$$

The following theorem indicates that forming simple linear beliefs in (10) can cause all the operating points in the achievable utility region to be CE.

Theorem 1: For Type II games, all the positive operating points in the utility region \mathcal{U} are essentially CE.

Theorem 1 establishes the existence of CE, i.e. for a particular $\mathbf{a}^* \in \mathcal{A}$, how to choose the parameters $\{\bar{s}_n, \bar{a}_n, \lambda_n\}_{n=1}^N$ such that \mathbf{a}^* is a CE. However, it neither tells us how these CE can be achieved and sustained in the dynamic setting nor clarifies how different belief configurations can lead to various CE.

We consider the dynamic scenarios in which users revise their reference points based on their past local observations over time. Let $s_n^t, a_n^t, \bar{s}_n^t, \bar{a}_n^t$ be user n 's state, action, belief function, and reference points at stage t , in which $s_n^t = \mu - \sum_{m=1}^N \tau_m a_m^t$. We propose a simple rule for individual users to update their reference points. At stage t , user n sets its \bar{s}_n^t and \bar{a}_n^t to be s_n^{t-1} and a_n^{t-1} . In other words, user n 's conjectured utility function at stage t is

$$u_n^t(\bar{s}_n^t(a_n), a_n) = a_n^{\beta_n} \cdot \left[\mu - \sum_{m=1}^N \tau_m a_m^{t-1} - \lambda_n (a_n - a_n^{t-1}) \right]. \quad (13)$$

Since we have defined the users' utility function at stage t , upon specifying the rule of how user n updates its action a_n^t based on its utility function $u_n^t(\bar{s}_n^t(a_n), a_n)$, the trajectory of the entire dynamic process is determined. The remainder of this paper will investigate the dynamic properties of the best response and Jacobi update mechanisms and the performance trade-off among the competing users at the resulting steady-state CE. In particular, for fixed $\{\lambda_n\}_{n=1}^N$, Section IV-C derives necessary and sufficient conditions for the convergence of the best response and the Jacobi update dynamics. Section IV-D quantitatively describes the limiting CE for given $\{\lambda_n\}_{n=1}^N$ and investigates how the parameters $\{\lambda_n\}_{n=1}^N$ should be properly chosen such that Pareto efficiency can be achieved.

C. Dynamic Algorithms

1) *Best Response:* In the best response algorithm, each user updates its action using the best response that maximizes its conjectured utility function in (13). Therefore, at stage t , user n chooses its action according to

$$a_n^t = B_n(\mathbf{a}^{t-1}) := \frac{\beta_n (\mu - \sum_{m \in \mathcal{N} \setminus \{n\}} \tau_m a_m^{t-1})}{\lambda_n (1 + \beta_n)} + \frac{\beta_n (\lambda_n - \tau_n) a_n^{t-1}}{\lambda_n (1 + \beta_n)}. \quad (14)$$

We are interested in characterizing the convergence of the update mechanism defined by (14) when using various λ_n to initialize the belief function \bar{s}_n .

To analyze the convergence of the best response dynamics, we consider the Jacobian matrix of the self-mapping function in (14). Let J_{ik} denote the element at row i and column k of the Jacobian matrix \mathbf{J} . The elements of the Jacobian matrix \mathbf{J}^{BR} of (14) are defined as:

$$J_{ik}^{BR} = \frac{\partial a_i^t}{\partial a_k^{t-1}} = \begin{cases} \frac{\beta_k (\lambda_k - \tau_k)}{\lambda_k (1 + \beta_k)}, & \text{if } i = k, \\ -\frac{\beta_i \tau_k}{\lambda_i (1 + \beta_i)}, & \text{if } i \neq k. \end{cases} \quad (15)$$

By analyzing the eigenvalues of \mathbf{J}^{BR} , the following theorem gives a necessary and sufficient condition under which the best response dynamics defined in (14) converges.

Theorem 2: For Type II games, a necessary and sufficient condition for the best response dynamics to converge is

$$\sum_{n=1}^N \frac{\tau_n \beta_n}{\lambda_n (1 + 2\beta_n)} < 1. \quad (16)$$

Remark 1: Theorem 2 generalizes the necessary and sufficient condition derived in [21], where users are assumed to be symmetric, i.e. $\tau_n = 1, \forall n$ and they adopt the Nash strategy by choosing $\lambda_n = \tau_n, \forall n$. Due to lack of symmetry, the derivation in [21] is not readily applicable to analyze the convergence of the best response dynamics. The proof of Theorem 2 instead directly characterizes the eigenvalues of the Jacobian matrix, and hence, provides a more general convergence analysis of the dynamic algorithms that allow users to update their actions based on their independent linear conjectures.

Remark 2: In Type II games, a locally stable CE is also globally convergent, which is purely due to the property of its utility functions specified in (4). From (15), we can see that all the elements in \mathbf{J}^{BR} are independent of the joint play \mathbf{a}^{t-1} . This is in contrast with Type I games considered in [15], where local stability of a CE may not imply its global convergence and the best response dynamics may only converge if the operating point is close enough to the steady-state equilibrium.

2) *Jacobi Update:* We consider another alternative strategy update mechanism called Jacobi update [22]. In Jacobi update, every user adjusts its action gradually towards the best response strategy. At stage t , user n chooses its action according to

$$a_n^t = J_n(\mathbf{a}^{t-1}) := a_n^{t-1} + \epsilon [B_n(\mathbf{a}^{t-1}) - a_n^{t-1}], \quad (17)$$

in which the stepsize $\epsilon > 0$ and $B_n(\mathbf{a}^{t-1})$ is defined in (14). The following theorem establishes the convergence property of the Jacobi update dynamics.

Theorem 3: In Type II games, for given $\{\tau_n, \beta_n, \lambda_n\}_{n=1}^N$, the Jacobi update dynamics converges if the stepsize ϵ is sufficiently small.

Remark 3: For any $\{\tau_n, \beta_n, \lambda_n\}_{n=1}^N > 0$, the Jacobi update mechanism globally converges to a CE as long as the stepsize is set to be a small enough positive number. In other words, the small stepsize in the Jacobi update can compensate for the instability of the best response dynamics even though the necessary and sufficient condition in (16) is not satisfied.

D. Stability of the Pareto Boundary

In order to understand how to properly choose the parameters $\{\lambda_n\}_{n=1}^N$ such that it leads to efficient outcomes, we need to explicitly describe the steady-state CE in terms of the parameters $\{\lambda_n\}_{n=1}^N$ of the belief functions. Denote the joint action profile at CE as (a_1^*, \dots, a_N^*) . From Equation (14), we know that

$$(\lambda_n + \beta_n \tau_n) a_n^* + \sum_{m \in \mathcal{N} \setminus \{n\}} \beta_n \tau_m a_m^* = \beta_n \mu, \forall n \in \mathcal{N}. \quad (18)$$

The solutions of the above linear equations are

$$a_n^{CE} = \frac{\beta_n \mu}{\lambda_n (1 + \sum_{m=1}^N \frac{\tau_m \beta_m}{\lambda_m})}, \forall n \in \mathcal{N}. \quad (19)$$

Based on the closed-form expression of the CE, the following theorem indicates the stability of the Pareto boundary in Type II games.

Theorem 4: For Type II games, all the operating points on the Pareto boundary are globally convergent CE under the best response dynamics if and only if

$$\sum_{n=1}^N \frac{\tau_n}{\lambda_n} = 1. \quad (20)$$

Remark 4: To achieve Pareto-optimality in the non-cooperative scenarios, users need to choose the belief parameters $\{\lambda_n\}_{n=1}^N$ to be greater than or equal to the parameters $\{\tau_n\}_{n=1}^N$ in the utility function $\{u_n\}_{n=1}^N$ and the summation of $\frac{\tau_n}{\lambda_n}$ should be equal to 1. Define user n 's conservativeness as $\frac{\tau_n}{\lambda_n}$, which reflects the ratio between the immediate performance degradation $-\tau_n \Delta a_n$ in the actual utility function and the long-term effect $-\lambda_n \Delta a_n$ in the conjectured utility function if user n increases its action by Δa_n . The condition in Equation (20) indicates that, to achieve efficient outcomes, the non-collaborative users need to jointly maintain moderate conservativeness by considering the multi-user coupling and appropriately choosing $\{\lambda_n\}_{n=1}^N$. By ‘‘moderate’’, we mean that users are neither too aggressive, i.e. $\lambda_n \rightarrow \tau_n$ and $\sum_{n=1}^N \frac{\tau_n}{\lambda_n} \rightarrow N$, nor too conservative, i.e. $\lambda_n \rightarrow +\infty$ and $\sum_{n=1}^N \frac{\tau_n}{\lambda_n} \rightarrow 0$. If more than one user plays the Nash strategy and choose $\lambda_n = \tau_n$, Equation (20) does not hold and the resulting operating point is not Pareto-optimal. Therefore, myopic selfish behavior is detrimental.

Using Jensen's inequality, we can verify that $\sum_{n=1}^N \omega_n \log \frac{u_n(\mathbf{a}^{CE})}{u_n(\mathbf{a}^{PB})} \leq 0$ and $\sum_{n=1}^N \omega_n \log \frac{u_n(\mathbf{a}^{CE})}{u_n(\mathbf{a}^{PB})} = 0$ if and only if $\omega_n = \frac{\tau_n}{\lambda_n}, \forall n$. Therefore, if a CE is Pareto efficient, user n 's conservativeness τ_n/λ_n corresponds to the weight assigned to user n in the weighted proportional fairness defined in (8).

As an illustrative example, we simulate a three-user system with parameters $\beta = [1.5 \ 1 \ 0.5]$, $\tau = [3 \ 4 \ 5]$, $\mu = 10$, $\omega_n = \frac{1}{3}, \forall n$. In this case, the joint actions and the corresponding utilities at NE and Pareto boundary are summarized in Table I. Fig. 1 shows the trajectory of the action updates under both best response and Jacobi update dynamics, in which $a_n^0 = 0.5$, $\lambda_n = \frac{\tau_n}{\omega_n}, \forall n$, and $\epsilon = 0.5$. The best response update converges to the Pareto-optimal operating point in around 8 iterations and the Jacobi update experiences a smoother trajectory and the same equilibrium is attained after more iterations.

E. Discussions

1) *Comparison between Type I and Type II games:* As mentioned before, the properties of Type I games have been investigated in the context of wireless random access [15]. Table II summarizes some similarities and differences between both types of games. First, the two algorithms exhibit different properties under the best response dynamics. In Type I games,

TABLE I
ACTIONS AND PAYOFFS AT NE AND PARETO BOUNDARY.

	User 1	User 2	User 3
a_i^{NE}	1.25	0.625	0.25
u_i^{NE}	3.4939	1.5625	1.25
a_i^{PB}	0.833	0.417	0.167
u_i^{PB}	3.8036	2.0833	2.0412

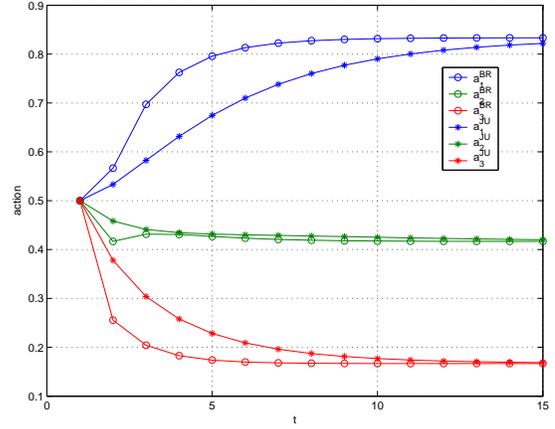


Fig. 1. The trajectory of the best response and Jacobi update dynamics.

the stable CE may not be globally convergent. However, the local stability of a CE implies its global convergence in Type II games. Second, it is shown in [15] that any operating point that is arbitrarily close to the Pareto boundary of the utility region of Type I games is a stable CE. Similarly, the entire Pareto boundary of Type II games is also stable. At last, different relationships between the parameter selection and the achieved utility at equilibrium have been observed for the two types of games. In particular, in Type I games, user n 's utility u_n is approximately proportional to the inverse of the parameter λ_n in its belief function. In contrast, in Type II games, if the CE is Pareto-optimal, the ratio τ_n/λ_n coincide with the weight ω_n assigned to user n in the proportional fairness objective function. In other words, based on the definition of proportional fairness [25], we know

$$\sum_{n=1}^N \frac{\tau_n (u'_n - u_n^*)}{\lambda_n u_n^*} \leq 0, \quad (21)$$

in which $(u'_1, u'_2, \dots, u'_N)$ is the users' achieved utility associated with any other feasible joint action and $(u_1^*, u_2^*, \dots, u_N^*)$ is the optimal achieved utility for problem (8) with $\omega_n = \tau_n/\lambda_n$ and $\sum_{n=1}^N \omega_n = 1$.

2) *Pricing Mechanism vs. Conjectural Equilibrium:* In order to achieve Pareto-optimality, information exchanges among users is generally required in order to collaboratively maximize the system efficiency. The existing cooperative communication scenarios either assume that the information about all the users is gathered by a trusted moderator (e.g. access point, base station etc.), to which it is given the authority to centrally divide the available resources among the participating users, or, in the distributed setting, users

TABLE II
COMPARISON BETWEEN TYPE I AND TYPE II GAMES.

Games	Best response dynamics	Stability vs. efficiency	Fairness vs. parameter selection
Type I	local stability \Leftarrow global convergence	stable at near-Pareto-optimal points	$u_n \propto \tau_n/\lambda_n$
Type II	local stability \Leftrightarrow global convergence	stable at the Pareto boundary	$\omega_n = \tau_n/\lambda_n$ at the Pareto boundary

exchange price signals (e.g. the Lagrange multipliers for the dual problem) that reflect the “cost” for consuming per unit constrained resources to maximize the social welfare and reach Pareto-optimal allocations. As an important tool, the pricing mechanism has been applied in the distributed optimization of various communication networks [12]. However, we would like to point out that, the pricing mechanism generally requires repeated coordination information exchange among users in order to determine the optimal actions and achieve the Pareto-optimality. In contrast, for the linear coupled communication games, since the specific structure of the utility function is explored, the CE approach is able to calculate the Pareto efficient operating point in a distributed manner, without any real-time information exchange among users. In fact, the underlying coordination is implicitly implemented when the participating users initialize their belief parameters. Once the belief parameters are properly initialized by the protocol according to (20), using the proposed dynamic update algorithms, individual users are able to achieve the Pareto-optimal CE solely based on their local observations on their own states and no message exchange is needed during the convergence process. Therefore, the conjecture equilibrium approach is an important alternative to the pricing-based approach in the linearly coupled games.

V. CONCLUSION

We derive the structure of the utility functions in the multi-user communication scenarios where a user’s action has proportionally the same impact over other users’ utilities. The performance gap between NE and Pareto boundary of the utility region is quantified. To improve the performance in non-cooperative cases, we investigate a CE approach which endows users with simple linear beliefs which enables them to select an equilibrium outcome that is efficient without the need of explicit message exchanges. The properties of the CE under both the best response and Jacobi dynamic update mechanisms are characterized. We show that the entire Pareto boundary in linearly coupled games is globally convergent CE which can be achieved by both studied dynamic algorithms without the need of real-time message passing. A potential future direction is to see how to extend the CE approach to the general linearly coupled games that are compositions of the basic two types and certain particular non-linearly coupled multi-user communication scenarios.

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